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Sampling-Period influence in performance and stability in Sampled-Data control systems

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ABSTRACT

As already verified experimentally in foregoing works [1, 2] the dimensioning of the sampling period in sampled-data systems (analog plant and digital controller) aiming principally the stability is a very important task due the increasing of the lawsuit of this kind of systems. However, the theory to study this nature of systems is not complete today. In this work we search for to give the initial steps for design of discrete controllers for sampled-data systems considering an expression for the aliasing.

INTRODUCTION

As shown in other works [1, 2], the problem of the influence of the sampling period (T) in the performance of sampled-data systems (analog plant and digital controller) is a real, serious and problematic phenomenon and very difficult for a theoretical analysis. In this work we are giving the initial steps to make a form of a theory that can explain this interference.

At the current literature [5, 6, 9, 16] we know that the Z-transform has its principal domain limited from $-\pi/T$ to $+\pi/T$. The designer must to be careful to know that this will be the frequency range that your controller will act at his simulations (see figure 1).

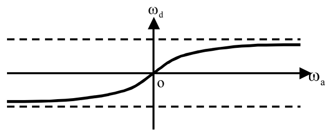

Figure 1: limited frequency range of Z-Transform.

However, there are infinite others frequency strips that cannot be ignored. The exponential mapping is a multivoc mapping: it is periodic on the imaginary axis and from this phenomenon borns the aliasing. From these aspects we can write:

$$G_T(z) = G_{*,T}(z) + \frac{1}{T}\sum_{\substack{k\neq 0\\ k=-\infty}}^{\infty} \frac{1-e^{-sT}}{s}.\frac{G(s)}{s}\Bigg|_{s=j\left(\omega_a - k\frac{2\pi}{T}\right)} \qquad \textbf{Eq 1}$$

Where, $G_T(z)$ = "true discrete"; $G_{*,T}(z)$= mapping done in the principal domain from $-\pi/T$ to $+\pi/T$ (is the result obtained with the step-invariance method), $\Delta G_*(z) = H_T(z) = \frac{1}{T}\sum_{\substack{k\neq 0\\ k=-\infty}}^{\infty} \frac{1-e^{-sT}}{s}.\frac{G(s)}{s}\Big|_{s=j\left(\omega_a - k\frac{2\pi}{T}\right)}$ = is the term that origins the aliasing.

We need that our discrete controller must be robust in face the aliasing disturbations. Note that different signals can force different frequency strips. For all the subset Θ of integer numbers it has a signal that exists only on the strips

$\left((2k+1)\frac{\pi}{T},(2k-1)\frac{\pi}{T}\right)$ with $k \in \Theta$. Where the set of all disturbances to $G_{*,T}(z)$ could be described by S_T :

$$S_T = \left\{ \frac{1}{T}\sum_{k\neq 0} \frac{1-e^{-sT}}{s} \cdot \frac{G(s)}{s} \bigg|_{s=j\left(\omega_a - k\frac{2\pi}{T}\right)} \quad ; \quad \forall k \in \Gamma \right\} \qquad \textbf{Equ. 2}$$

Note that in Equation 2 the discrete time k.T permits to do combinations such that, if Γ is the set of indices of all possible combinations that can occur in discrete-time (combinations between the different strips), then Γ is the class of all subsets of N-{0}.

STABILITY BOUNDARIES

Assume that $C_{K,T}(z) = [X(z)-K(z)N(z)]^{-1}[Y(z)+K(z)D(z)]$ is a controller stabilising $G_T(z) = N(z)D^{-1}(z)$ associated to the proper stable rational function K(z). Note that N(z) and D(z) are functions of T. By usual theory, it is possible to find the most general class of perturbations in the form:

$$S_{T,K} = \left\{\Delta H(z) : \|\Delta H(z)\| \leq \lambda\right\} \qquad \textbf{Equ. 3}$$

such that $C_{K,T}(z)$ is still a stabilising controller for all G(z) generated by the perturbation og $G_T(z)$ by some transfer function on $S_{T,K}$. Denote this maximal class by the same symbol, $S_{T,K}$. The problem can be described as: there is a proper stable rational function K(z) such that $S_T \subset S_{T,K}$? That is, there is a stabilizing controller robust for plant perturbations free on the aliasing perturbation set S_T. In such hyphothesis, what is the set oif K(z) having this property? Finally, how the aliasing stability property and the consequent stabilising set are dependent from T?

The sense of this last question is directed to solve the problem of maximize the sampling period T, maintaining stability and some performance level described in function of the parameter K(z). Quadratic performance criteria are suitable for this problem.

The complexity of this problem is related to the dependence of N(z) and D(z) from T. In a geometrical representation,

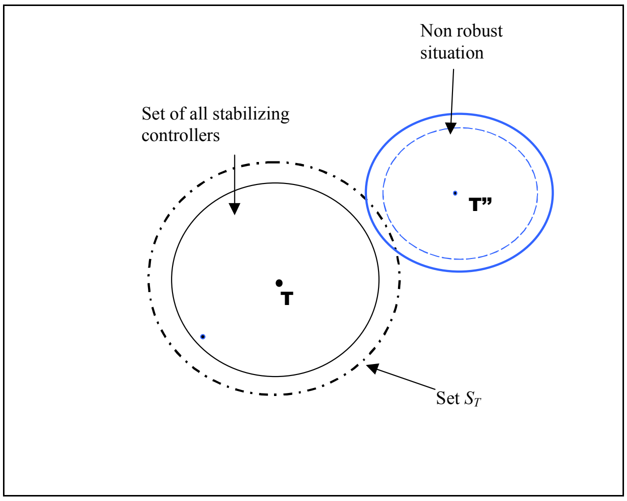

Figure 2: Stability and instability situations..

where we can see that the center and the radius of each ball is T-dependent.

NUMERICAL RESULTS

This numerical example simulates a position sampled-data of a servo harmonic-oscillator with discrete-control done in [1]. In the figure 3 we can see the sampled data control implemented. In figures 5, 6 and 7 we can see respectively: the case obtained for asymptotically stability (using T=0.1 sec), a strange limit-cycle (using T=2 sec) and the instability detection (for T=0.5 sec).

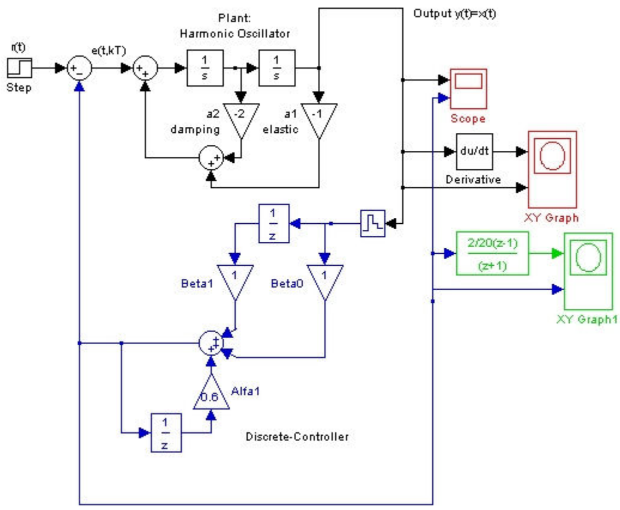

Figure 3: sampled-data control simulation circuitry.

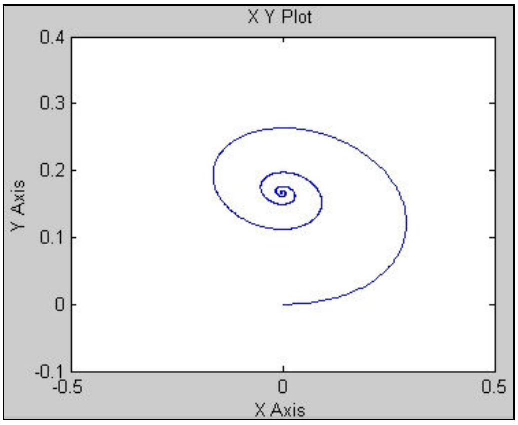

Figure 4: Stable case (T=0.1 sec).

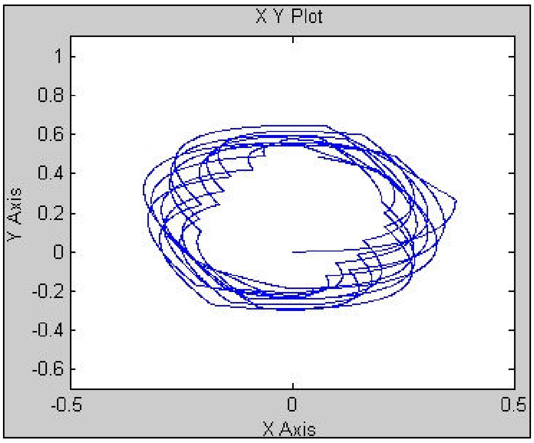

Figure 5: Strange Limit-Cycle (T=2 sec).

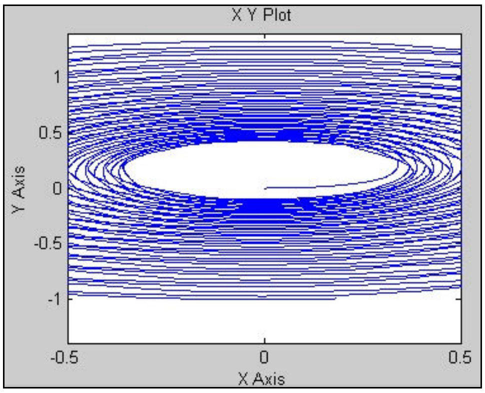

Figure 6: Instability detection (T=0.5 sec).

CONCLUSION

In this work we can see that varying the sampling period T it is possibly to have a aliasing robust stabilizing controller for some periods, and a not robust controller if the period is too large. A formalization of this phenomenon is given as a robust control problem where the perturbed set is changed with the sampling period T not only in its radius but also in its center – and there is the greatest formal difficulty to solve the problem. This problem is under investigations, as its application to sampled-data orbit control.

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