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Neural Network Adaptive Wavelets for Predictions of the Northeastern Brazil Monthly Rainfall Anomalies Time Series

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ABSTRACT

Neural networks were used to predict the anomalies of the time series of monthly rainfall of the Northeastern Region of Brazil. The forecasts made using a feedforward network with backpropagation algorithm from the original data were not satisfactory. We have therefore tried to combine two advanced methods, Wavelet Transform and Neural networks. Three more types of neural networks were used. The selected neural networks include the Time Delay Neural Networks (TDNN), Radial Basis Functions (RBF) network and Neural Network Adaptive Wavelet. All networks were implemented in neural network simulator SNNS. The Neural Network Adaptive Wavelet was implemented by changing the standard sigmoidal nonlinearities to wavelet nonlinearities in the neurons. We compare the results obtained with unfiltered and filtered data. Using data obtained by filtering the wavelet transform coefficients significantly improved the results for all networks. The combination of TDNN with wavelet filtered data gave the best results.

Keywords: Neural networks, rainfall, time series prediction, wavelet transform, time series filtering.

1. INTRODUCTION

The Northeastern Region of Brazil is known to have one of the "earth's problem climates" due to its geography (1-18°S, 35-47°)[1]. Many experiments for prediction of the rainfall time series over this region have been carried out, most of them considering the precipitation time series as a linear dynamic system[2,3,4]. To represent this region, the monthly rainfall time series of the Fortaleza City from years 1849 to 1984 was used. For this series, feed-forward neural networks and the conventional backpropagation algorithm were implemented. The preliminary results for reconstruction and prediction, which used the raw data, were not acceptable. In this research, we tried to combine Wavelet Transform with Neural Networks. Provided with the filtered series, the neural networks achieved better reconstruction and prediction results than when it used the original data. The selected neural networks include Time Delay Neural Networks (TDNN), Radial Basis Functions (RBF) network and Neural Network Adaptive Wavelets. All were implemented in neural network simulator SNNS. We compare the results obtained from the four methods, the first with unfiltered data and the others with filtered data. Using data obtained by filtering the wavelet transform coefficients significantly improved the results for all networks. The combination of TDNN with wavelet filtered data achieved the best results.

The paper is organized as follows: the methodology is summarized in section two, a brief discussion of Wavelet Transforms, Neural Networks, Neural Network Adaptive Wavelet and SNNS is given; section three describes the data and its filtering technique using the Third-Order Coifmann Wavelet Transform. Section four analyses the results of the neural network reconstruction and predictions. We specially analyzed the differences in results according to structure, algorithm and type of data (original and filtered data). Section five contains our conclusions about the results.

2. METHODOLOGY

2.1. The Wavelet Transform

The Wavelet Transform (WT) is a linear transform that is obtained by convoluting a given signal $s(t)$, with the translated and dilated versions of a single function ϕ(t) called the mother wavelet:

$$T_\phi(a,\tau)=\frac{1}{\sqrt{a}}\int_{-\infty}^{\infty}\phi^*_{a\tau}\left(\frac{t-\tau}{a}\right)s(t)\,dt$$

where

$$\phi_{ab}(t)=a^{-1/2}\phi\left(\frac{t-b}{a}\right)$$

is called a wavelet. a is the dilation or scaling parameter and b the translation parameter.

Only a function ϕ(t) whose Fourier Transform satisfies the admissibility condition:

$$\int_{-\infty}^{\infty}\frac{|\phi(\omega)|^2}{|\omega|}\,d\omega<\infty,$$

can be used as the mother wavelet. For a function $\phi(t) \in L^2(R)$, this requires that the mean value of the function is zero. In practice, in addition to admissibility, it is desirable to impose other constraints on the function ϕ (t). Greater details can be found in Daubechies' book[5].

If $a \in R^+$ and $b \in R$, the WT is called the continuous Wavelet Transform. When a and b are computed only for a discrete set of values, the WT constitutes a frame. A most natural and efficient discretization is $a = 2^{-j}$ and $b = n\ 2^{-j}$ If ϕ satisfies some additional conditions, the wavelets form an orthogonal basis of $L^2(R)$. The decomposition of a signal in terms of these basis is called multiresolution analysis[5,6] and is done in O(N) operations for a data of size N.

The WT permits a local analysis in both the time and frequency domains. For each daughter-wavelet $\phi_{ab}(t)$ the temporal localization is determined by b and the frequency localization by a. The values of $T_\phi(a,\tau)$ express, then, the decomposition of signal $s(t)$ at a specific position τ and at a specific scale a. The set of these parameters' values provide a bidimensional representation of the signal in time and frequency.

2.2. Neural networks

Prediction of time series is an important application of neural networks. The problem of predicting time series using this method has been studied by many people[7,8]. Neural networks were found to be useful and competitive with the best recent approximation methods[9,10,11,12]. To predict the precipitation of Fortaleza City from the data, four kinds of neural networks were used: feed-forward networks with Backpropagation, Time Delay Neural Networks (TDNN), Radial Basis Functions (RBF) network and Neural Network Adaptive Wavelets. The Backpropagation, TDNN and RBF algorithms were implemented in the neural networks simulator SNNS. The Neural Network Adaptive Wavelets were implemented changing the standard sigmoidal nonlinearities to wavelet nonlinearities in the artificial neurons within the SNNS. Here we give a brief description about first three neural network models, Neural Network Adaptive Wavelets are shown in the next section.

2.2.1. Backpropagation network

The most popular network is the feed forward network with backpropagation learning law[13,25]. The input values of time series $x(t-1), x(t-2),...x(t-d)$ are received through d input units, which simply pass the input forwards to the hidden units u_j, $j = 1,2,...,q$. Each connection performs a linear transformation determined by the connection strength w_{ij}, so the total input for hidden unit u_j is $\sum^{d}_{i=1} w_{ij}\, x(t-i)$. Each unit performs a nonlinear transformation on its total input, producing the output:

$$u_i = \Psi(W_{0j} + \sum_{i=1}^{d} W_{ij} X(t-i))$$

The activation function Ψ is the same for all units. Here, Ψ is a sigmoid function with limiting value 0 and 1 as $u_j \rightarrow -\infty$ and $u_j \rightarrow +\infty$ respectively:

$$\Psi(u_j) = \frac{1}{(1+e^{-u_j})}$$

The hidden layer outputs u_j are passed along to the single output unit with connection strength β_j, which performs an affine transformation on its total input. Then, the network's output $x(t)$ can be represented as:

$$x(t) = \beta_0 + \sum_{j=1}^{q} \beta_j . \Psi . \left(w_{0j} + \sum_{i=1}^{d} w_{ij} . x(t-i) \right)$$

for d inputs and q units in the hidden layer.

2.2.2. Time Delay Neural Networks --- TDNN

The Time Delay Neural Network is a layered network in which the outputs of a layer are buffered several time steps and then fed fully connected to the next layer[14,15]. The activation of a unit is normally computed by passing the weighted sum of its inputs to an activation function, usually a threshold or sigmoid function. For TDNN this behavior is modified though the introduction of delay[16]. Training in this kind of network is performed by a procedure similar to backpropagation, that takes the special semantics of coupled links into account. To enable the network to achieve the desired behavior, a sequence of patterns has to be presented to the input layer with the feature of interest shifted within the patterns.

2.2.3. Radial Basis Functions (RBF) network

The principle of radial basis functions network derives from the theory of functional approximation[17,18,16]. For N pairs (x_i, y_i) ($x_i \in \Re^n$, $y \in \Re$), there is a f function:

$$f(x_i) = \sum_{i=1}^{k} c_i . h(|x_i - t_i|)$$

h is the radial basis function and t_i and the K centers which have to be selected. The coefficients c_i are also unknown at the moment and have to be computed. x_i and t_i are elements of an n-dimensional vector space. h is applied to the Euclidean distance between each center t_i and given argument x_i.

For a fully connected feedforward network with n input, K hidden and m output neurons, the activation of output neuron k on the input $x_i = x_1, x_2, \ldots, x_n$ to the input units is:

$$o_k(x_i) = \sigma\left(\sum_{i=1}^{k} c_{j,k} . h\left(. |x_i - t_i| . p_j \right) + \sum_{i=1}^{n} d_{i,k} . x_i + b_k \right)$$

where the coefficients $c_{j,k}$ represent the links between hidden and output units. The shortcut connections from input to output are done by $d_{i,k}$. b_k is the bias of the output units and p_j is the bias of the hidden neurons which determines the exact characteristics of the function h .

2.3. Neural network adaptive wavelet

A signal $x(t)$ can be approximated by daughters of a mother-wavelet $h(t)$ according to

$$x(t) = \sum_{k=1}^{k} w_k . h\left(\frac{(t - b_k)}{a_k} \right)$$

where the w_k , b_k and a_k are the weight coefficients, shifts and dilation for each daughter-wavelet. This approximation can be calculated using a neural network[19,20,26,27] if one uses radial basis wavelets . Since some symmetric wavelets can be used in radial basis functions (RBF) networks, these networks are called Neural Network Adaptive Wavelets. The network parameters w_k, b_k and a_k can be optimized by minimizing an energy function.

In this work, Morlet wavelet and Derivative-of-Gaussian wavelet were used. The general form of the Morlet function is:

$$h(t) = \cos(\rho t) \exp(-t^2/2),$$

and the form of the Derivative-of-Gaussian function is:

$$h(t) = -2x/\rho^2 \exp(-x^2/(2*\rho)).$$

2.4. Stuttgart neural network simulator -- SNNS

The Stuttgart neural network simulator (SNNS) is a simulator for neural networks developed at the Institute for Parallel and Distributed High Performance Systems (IPVR) at the University of Stuttgart. The goal of the project is to create an efficient and flexible simulation environment for research and application of neural networks[16]. In this paper, SNNSv4.0 was used. It has been developed by IPVR since 1995. As described above, Backpropagation, TDNN and RBF algorithms were implemented in the SNNS.

The Morlet and Derivative-of-Gaussian wavelets were implemented by authors changing the standard sigmoidal nonlinearities to Morlet or Derivative-of-Gaussian wavelet nonlinearities in the artificial neurons within the SNNS.

3. THE DATA AND WAVELET FILTERING

3.1. The data

The Northeastern Region of Brazil (NEBR), whose area is of approximately 1 million Km^2, and whose population is of many dozen million persons, has a climate characterized by strong interannual variations of precipitation. This has dramatic consequences on the inhabitants of the regions. Though its east coast has an average 2000 mm of precipitation each year, parts of its interior have an average smaller than 400 mm[21].

In spite of the relevance of the study of precipitation variability in NEBR, it is not easy to be conducted, due to the inexistence of enough data series sufficiently long, with more than a century of measurements. One of the few exceptions is the precipitation data series from Fortaleza (4° S, 39° W), a coastal city of the Brazilian Ceará State, which starts in 1849. This is certainly the main reason why this series has been intensely investigated. There, as Nobre and others[3] explain, the rainy season is from January to June, a period in which precipitation is, in average, nearly 90% of the annual total. The months in which precipitation is greater are March, April and May, and one can define an hydrological year from November to next October.

For this data, Teixeira et al.[2], Nobre et al.[3], Kane and Trivedi[4] tried to prove existence of periodicity in it. When they applied spectral methods in the analysis of the series, they seemed to agree on existence of statistically significant periodicity, with periods of approximately 13 and 26 years. However, Kantor[22], using an auto-regressive process for prediction whose coefficients were calculated through Burg's maximum entropy method, showed that the residual error in the prediction is big when compared to the variance of the data. Therefore the previsibility of the data series is low, according to him. Nobre and others[3] also emphasize the degree of uncertainty in the prognostic of droughts in NEBR through the method of periodicity, because the two statistically significant periodicity explain only 24% of the series variance. Hastenrath and others[23] adapted a non-spectral statistical procedure to monitor and predict droughts in NEBR through the method of precipitation prediction for the March-April and March-September periods. To do that they tested meteorological variables which were dynamically plausible for a stepwise multiple linear regression scheme. The regression equations were then used to make predictions for each of the years from 1958 to 1972 in terms of a precipitation seasonal index for the NEBR as a whole. However, the predictions only explained from 41% to 62% of the variance of the observed precipitation. Another study with similar methodology was developed by Hastenrath and Greischar[24], in which analysis using empirical orthogonal functions (EOFs) was applied to form indexes of meridional wind component and of SST. These series formed the input for stepwise multiple regression models both for an experimental neural network and a linear discriminate analysis. In this way, the authors could explain 50% to 75% of the precipitation interannual variation. All these methods are linear.

In contrast to the linear approaches mentioned above, we are using nonlinear regression models, nonlinear neural networks, which we believe can be more powerful.

3.2. Filtering of data using Wavelet Transform

The original data is very noisy but has a very strong annual component. To suppress the noise and to get the annual variation wavelet filtering was used. The Multiresolution Analysis (MRA) was done on the data using Daubechies wavelets, Coiflets (Coifmann wavelets) and Symmlets (Symmetric wavelets). The type of wavelet used did not significantly alter the results and hence only the results using Coiflet3 are presented.

The MRA was done on 1632 points up to 5 octaves. Detail signals in the first two octaves have only high frequencies and have no periodic components. The next three octaves contain almost all the periodic components. Hence the Inverse WT was computed by setting the first two detail signals to zero. Fig. 3.1 shows the detail signals. Of course, because of the loss of high frequency data the reconstructed signal is not the same as the original. Fig. 3.2 shows the original and filtered signals. In this study, since our interest was more in forecasting the relative increase/decrease in the amount of rainfall and not its actual value this loss of high frequency was not very important.

Currently we are investigating the use of stationary wavelets and other more sophisticated thresholds rather the hard threshold as presented here. Also the appropriateness of using different wavelets for different scales of the same signal is under investigation.

4. RECONSTRUCTION AND PREDICTION USING NEURAL NETWORKS

To reconstruct and predict the time series, we studied the following cases:

1) Choice of neural network structures;
2) Comparison of four neural network models;
3) Comparison between original and filtered data.

4.1. Choosing neural network structures

The first step of our analysis is to choose the best neural network structure for reconstruction and prediction of our time series. We used three different feed forward network structures and one TDNN structure. Two group of data (1024-point) were used: original data and filtered data using only detail signals with periodic components (w4).

We use the notation *input units : hidden units : hidden units : ... : output units* to describe the structure of the network. For the feed forward network, the three structures are: 1:10:1, 6:12:12:1 and 12:24:24:1. For the TDNN, the structure is 36:9:1. Both feed forward network and TDNN are implemented with Backpropagation algorithm. The simulation was executed in the SUN SparcSt. 10. The definition of the simulation error is described by Zell and others[16]. SSE means the sum of squared errors and MSE means the mean square error. The summary of the network simulation results is shown in Table 4.1.

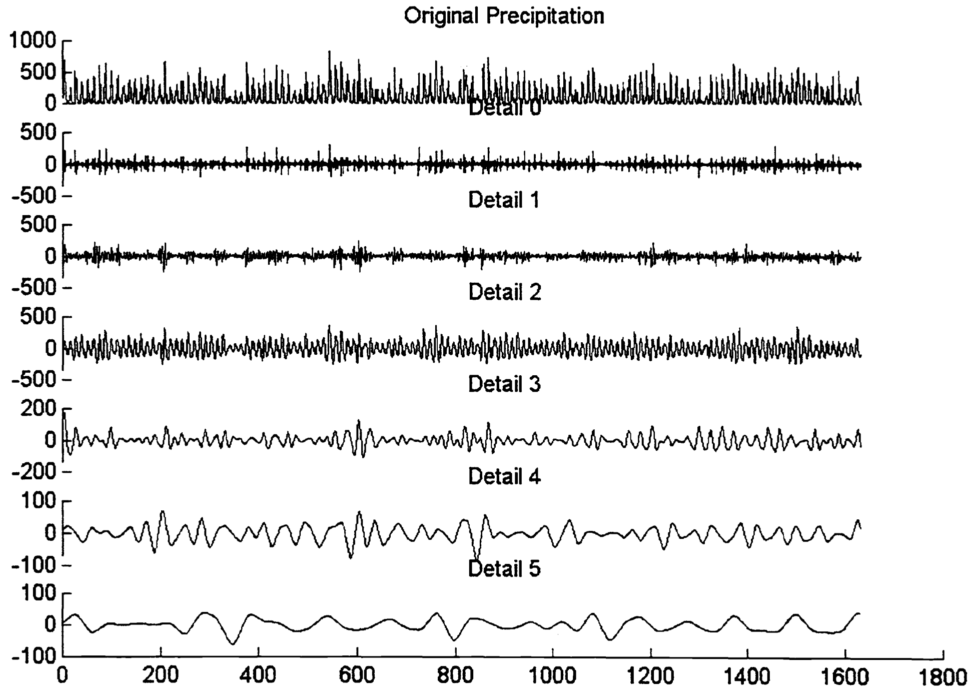

Fig. 3.1 Detail signals filtered by Wavelet Transform

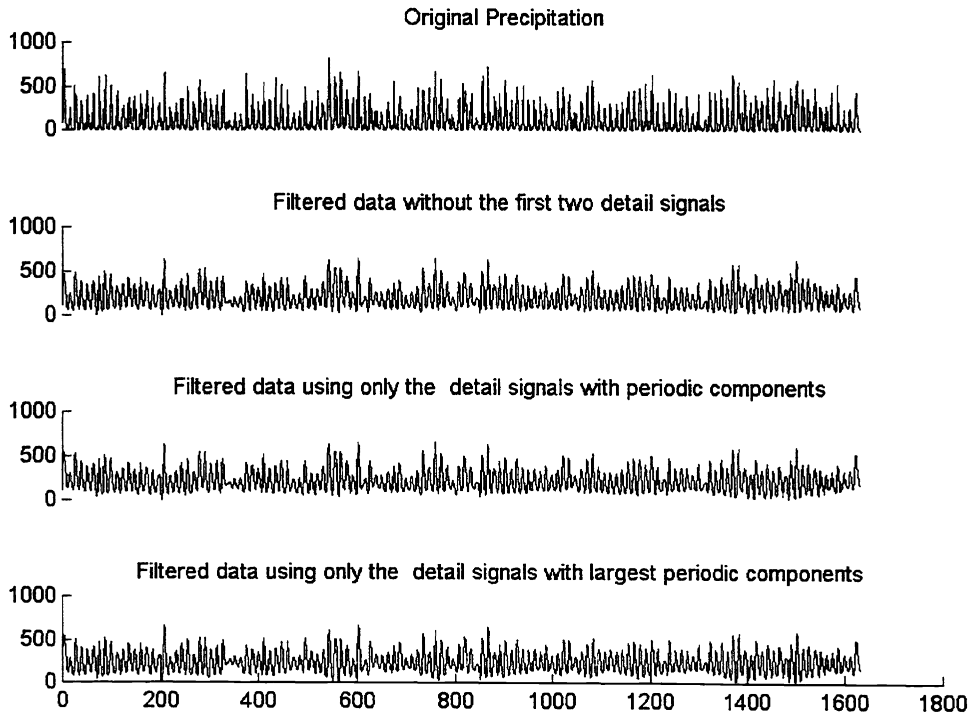

Fig. 3.2 Original and filtered signals

Table 4.1 Results from four neural network structures

Net structure	Data	Training cycles	Training time	SSE	MSE
1:10:1	Original Filtered w4	1000	8 min.	21.59 8.07	0.0211 0.0079
6:12:12:1	Original Filtered w4	1000	12 min.	17.23 3.58	0.0169 0.0035
12:24:24:1	Original Filtered w4	1000	52 min.	14.95 1.00	0.0148 0.0010
36:9:1 (TDNN)	Original Filtered w4	1000	6 min.	3.66 0.93	0.0036 0.0009

Table 4.1 shows that the simulation error is dependent mainly on the structure of the network. Considering the training time and the precision, 12:24:24:1 was considered to be the best structure of feedforward network for our intention. For TDNN we used the shown structure, whose precision in the simulation was acceptable.

4.2 Comparison of four neural network models

As we mentioned above, four kinds of neural network models were used to reconstruct and predict the time series. Table 4.2 gives the summary of the simulation results of these four models. 1024-point filtered data using only detail signals with periodic components (w4) were used.

From the Table 4.2, we can get the following results. For the filtered data w4, according to the training time (6 minutes for 1000 cycles), the TDNN has the best learning algorithm. After 1000 training cycles, the SSE is 0.93 and after 20000 training cycles, the SSE is 0.56. It is necessary to mention that even continuing to train the net, the SSE does not change. According to the simulation error, after 1000 training cycles, the SSE is 0.25 using RBF network. The simulation result for Neural Network Adaptive Morlet Wavelet was not so good. After 1000 training cycles, the SSE is 8.08, even after 6000 training cycles the SSE is bigger than for the other networks, 3.07. We also implemented the Derivative-of-Gaussian wavelet as the nonlinearity of the neural network, but the simulation result was not conclusive.

Table 4.2 Comparison among four neural network models

Net model	Data	Training cycles	Training time	SSE	MSE
Backpropa gation	Original Filtered w4	1000	52 min.	14.95 1.00	0.0148 0.0010
Time Delay Network	Original Filtered w4	1000	6 min.	3.66 0.93	0.0036 0.0009
Radial Basis Functions	Original Filtered w4	1000	20 min.	7.98 0.25	0.0079 0.0003
NN Adaptive Wavelet	Original Filtered w4	1000	20 min.	15.18 8.08	0.0150 0.0079

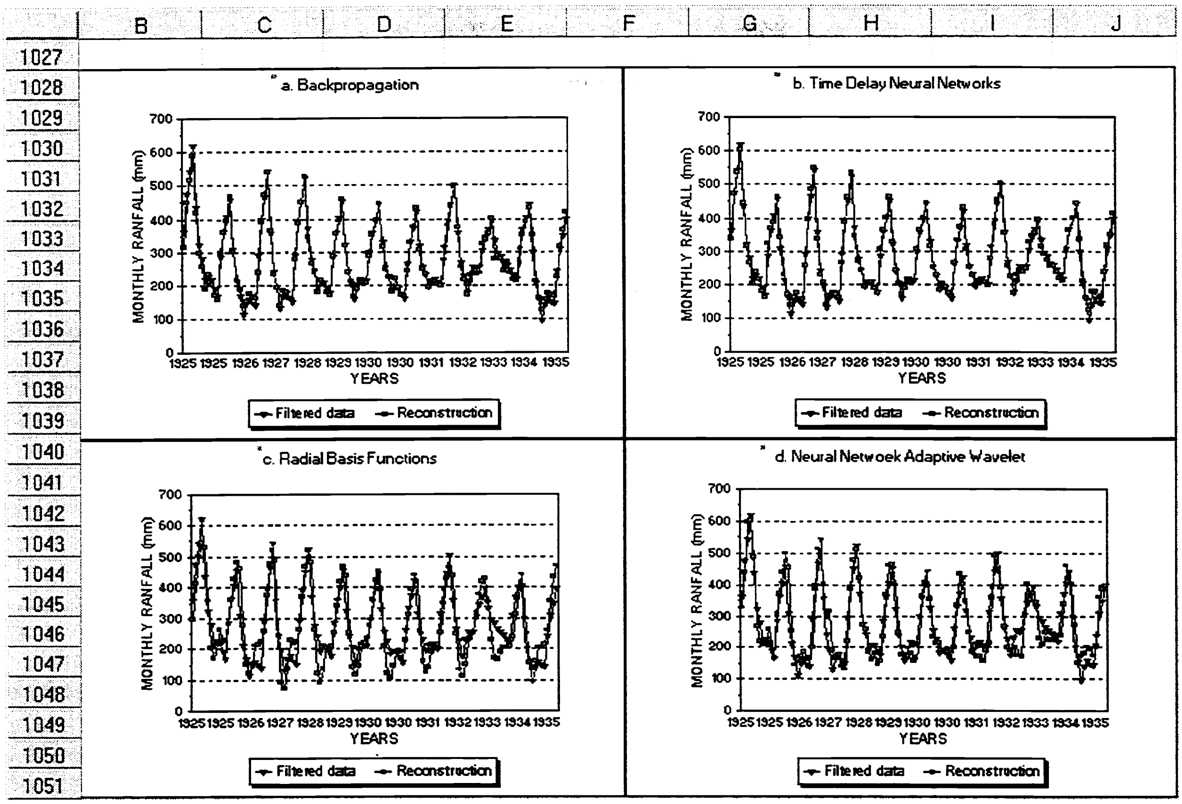

Fig. 4.1 Reconstruction of filtered signal (w4) using four neural network models
a. Backpropagation; ***b.*** TDNN; ***c.*** RBF; ***d.*** Neural Network Adaptive Wavelet.

Figure 4.1 shows the reconstruction of the filtered signal w4 (from 1925 to 1935, the last 120 data of 1024-point filtered signal) using four neural network models: ***a.*** Backpropagation; ***b.*** TDNN; ***c.*** RBF; ***d.*** Neural network adaptive Wavelet. The figure ***b*** shows TDNN best reconstructs w4, specially in some changing points.

Note that the reconstruction appeared to track correctly the tendency of extreme events to occur, though there could be small delay in their detection. Since the network is functioning as a filter, the error in the prediction in principle is greater than the error in the reconstruction/estimation, as can actually be verified in our graphs. Thus this small problem in the prediction is expected.

4.3.Comparison of prediction using original and filtered data

To analyze the effect of the filtrage using Wavelet Transform in prediction, we used TDNN to simulate four types of data. Now we used all available data (1632 points). For TDNN, 1620 data were used to train the network (20 of them were used to evaluate the trained network). The trained network predicted the next 12 data (12 months). The data series are of four types: 1) Original data; 2) Filtered data without the first two detail signals (s4); 3) Filtered data using only detail signals with periodic components (w4); 4) Filtered data using only detail signals with largest periodic components (w6); We chose to use w4 as time series for prediction because of its smaller variance, as indicated by Table 4.3.

Table 4.3 Comparison of types of data series using TDNN

Signal	Training cycles	SSE	MSE
Original	2000	20.1529	0.01289
Filtered s4	2000	0.3123	0.00020
Filtered w4	2000	0.28999	0.00019
Filtered w6	2000	0.2924	0.00019

Figure 4.2 shows the prediction for the year 1984 and reconstruction using the data from years 1976--1983. This year was chosen because it was very rainy. We used the TDNN on the four data series: ***a.*** Prediction from original data; ***b.*** Prediction from filtered data s4; ***c.*** Prediction from filtered data w4; ***d.*** Prediction from filtered data w6.

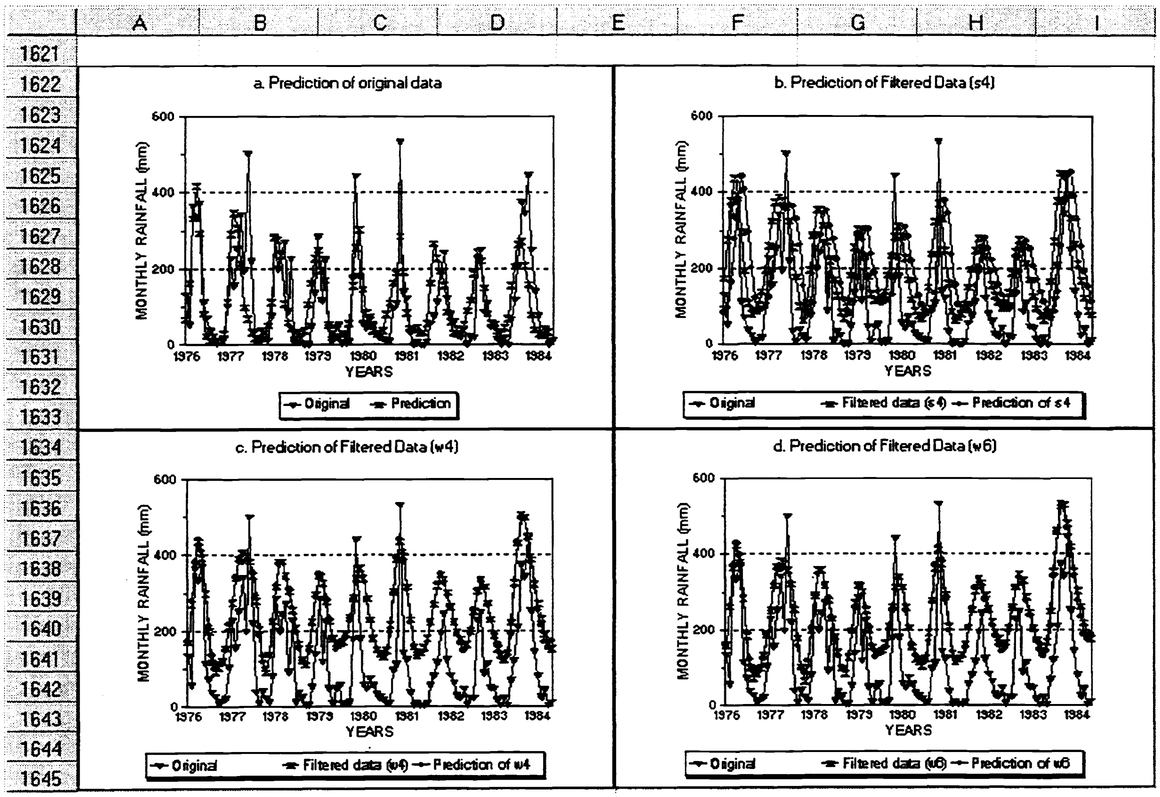

Figure 4.2 Reconstruction and prediction (1984) of four series using TDNN:
a. Original data; ***b***. Filtered data s4; ***c.*** Filtered data w4; ***d.*** Filtered data w6.

Table 4.4 shows the prediction of the precipitin of year 1984 using TDNN. We only show the prediction of the mean precipitation of the next three months using original and filtered w4 data. The prediction results were very clear. For original data, the relative error in prediction of the mean precipitation for the rainy season Jan.-Feb.-Mar. was 0.11, for Apr.-May.-Jun. was 0.39, for Jul.-Aug.-Sept. was 0.38 and for Oct.-Nov.-Dec. was 0.64. When using filtered data w4 the relative errors of prediction for the whole year were less than 0.05.

Table 4.4 Prediction of 1984's precipitation using TDNN

Data series	Period (1984)	SSE of trained net	Mean precipitation	Prediction	Related error
Original data	Jan,Feb,Mar	20.1529	233.33	207.48	0.11
	Apr,May,Jun		345.67	212.24	0.39
	Jul,Aug,Sep		80.66	49.71	0.38
	Oct,Nov,Dec		17.00	27.88	0.64
Filtered data w4	Jan,Feb,Mar	0.28999	420.59	415.31	0.01
	Apr,May,Jun		445.66	439.66	0.01
	Jul,Aug,Sep		274.05	259.64	0.05
	Oct,Nov,Dec		164.76	163.98	0.00

5. CONCLUSIONS

In this paper we used the neural network and wavelet transform methods to process Fortaleza City's precipitation time series (1849 - 1984). First, we used the Third-Order Coifmann Wavelet Transform to filter the data. Then, four neural network models were implemented to reconstruct and predict the time series. The results lead us to the following conclusions:

1) For the noisy height time series, precipitation of Fortaleza City, a wavelet transform was used to filter the data. The prediction made using this transformed input with the network was superior to that using the original data. However, the prediction results must be interpreted in terms of the filtered data, not in terms of the original data, which can make significant difference. In our case the filtering is justified because we are interested in predicting relative changes in precipitation, not absolute values.

2) The Time Delay Neural Network was an efficient model for reconstruction and prediction of time series. For Fortaleza City's precipitation time series, the relative error of prediction was less than 0.05 for filtered data w4;

3) The Radial Basis Function network model appears to have great potential for time series reconstruction. When used to reconstruct the filtered Fortaleza City's precipitation time series w4, the SSE was just 0.25 after 1000 cycles of training, less than that of all other three models;

4) The Neural Network Adaptive Wavelet appears to be a useful model for our interests. In this work, we just began to test the utility of the model. We implemented Morlet and Derivative of Gaussian wavelet in the Radial Basis Function model. Only for the Morlet wavelet did it achieve acceptable results (as Table 4.1 shows). Still, we intend to continue testing its usefulness using these two wavelets.

In addition to the above comments, we believe the influence of the El Niño/Southern Oscillation (ENSO) and/or Sea Surface Temperature (SST) on the precipitation of Northeastern Region of Brazil is another interesting study. We intend to try examining these influences in the future using neural networks.

6. ACKNOWLEDGMENTS

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