

# A STUDY OF INTRASEASONAL OSCILLATIONS ALONG THE WEST COAST OF SOUTH AMERICA USING WAVELET ANALYSIS

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Previous studies has shown that oceanic, poleward propagating low-frequency perturbations off the west coast of South America are linked with equatorial wave dynamics, especially during El Niño events, and can have a significant influence in modulating the upwelling system and primary productivity in the region, which is one of the most productive all around the world. An appropriate tool to analyse the spectral components of time series must let to the identification of the passage of such waves. Wavelet transform seems to be the most appropriate method to analyse signals that contain nonstationary power at many different frequencies.

In this study, wavelet transform (WT) and cross wavelet transform (XWT) was applied for analysing long time series of sea level and alongshore wind stress component to identify intraseasonal variability along off South America and its remote or local forcing. Also, it was used temperature and salinity profiles from oceanographic cruise realized during 1992 (El Niño year) to estimate properties of the coastal trapped waves with simple theoretical models.

It was observed for El Niño years (83, 87, 92, 97 e 98) the existence of intraseasonal oscillations with periods between 20-90 days along off coast comprised between 02°S e 27°S of latitude. Also, it was observed oscillations with strong correlation between sea level and local wind, with periods around of 10 days, could be associated with "coastal lows", coastal trapped waves in the atmosphere. These oscillations were more intense during El Niño 91-92. At the peak of the events El Niño 91-92 e 97-98 (at the beginning of 1992 and 1998) between 6°S e 12°S, it was observed perturbations probably associated with remotely forced internal kelvin waves with periods between 6 and 11 days (only for El Niño 97-98) with phase velocities between 160-260 km/day. In the region comprised between 12°S e 15°S and for two events El Niño, probably barotropic shelf waves propagated southward with velocities between 110 and 150 km/day and with periods between 30 e 50 days. In 1997, for this region, seems also that occurred baroclinic kelvin waves with period between 10 and 20 days, and velocities approximately of 200 km/day.

## 1. INTRODUCTION

The South American west coast its known by its very high primary productivity and also because this region is exposed to El Niño phenomenon, wich alters all the physical-biological ecosystem. Besides upwelling and the presence of poleward currents, the Peru-Chile Current system share a number of attributes with others boundary currents, including the presence of coastal trapped waves (CTWs) (Strub et. al., 1998). Several studies has detected low frequency oscillations, wich propagate poleward. Smith (1978), using superficial sea temperature and currents data along the peruvian coast during the 1976-1977 period, observed energy fluctuations at daily weekly scales. Poor correlations between the energy these fluctuations is about 200 km/day. Romea and Smith (1983), studing the same period but with sea fluctuations and local wind were found, instead high correlations with winds located at several hundred of kilometers off the coast. The phase speed of level data. They found low frequency fluctuations with periods longer than 4 days moving poleward with phase velocities correspondig to baroclinic kelvin waves (180-260 km/day). Brink (1982), analysing sea level and alongshore current speed component data

along the coast on the basis of the coastal trapped waves theory, suggest that the fluctuations at the 5-10 days band are due to equatorial free waves originating equatorward of 5°S. Clarke (1983), suggests that waves with periods of one to two weeks are forced by mixed Rossby-Gravity waves equatorially trapped and that travels to east. Enfiled (1987) found a significant intraseasonal variability off the coast of Peru with 40-70 days period. These oscillations are more noticed at the southern hemisphere spring-summer time and at the onset of El Niño, derivate from the baroclinic equatorial waves. Shaffer (1997) studied remotely forced austral trapped waves. As these waves can strongly modify the source of upwelled waters, thes could has a significant influence on the pelagic ecosystem along chilean coast (also along peruvian coast), at least during El Niño. In tha present work, a wavelet transform method was applied to identify statistically significant oscillations during the occurrence of some of the last El Niño events (1982-1983, 1991-1992 and 1997-1998) along off South American west coast. Properties of remotely forced perturbations were compared with simples theoretical models of the coastal trapped waves (barotropic shelf waves and baroclinic kelvin waves).

## 2. Data and Methodology

The data used in the present study correspond to historical time series from Sea Level Center of NODC and wind stress from IFREMER, with temporal resolutions of one day and seven days,

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respectively. In Table No. 1 is shown stations were used. We assumed that the data obey a normal distribution and was used anomalies with respect to the sample mean. For the sea level oscillations associated with the Nyquist frequency (period of 2 days) and the annual cycle variability were removed of the time series. A linear interpolation from the weekly stress wind data set was performed with the aim to obtain daily data. Also, a transform of rotation was applied to obtain the alongshore wind stress component.

**Table 1.** Locations of hydrographic and meteorological stations.

Station	Latitude (S)	Longitude (W)
La Libertad (Ecuador)	02°12'	80°55'
Lobos de Afuera	06°56'	80°43'
Paita	05°05'	81°10'
Santa Cruz (Ecuador)	00°45.3'	90°18.8'
Callao	12°03'	77°09'
Pisco	13°25'	76°08'
San Juan	15°22'	75°12'
Matarani	17°00'	72°07'
Arica	18°28'	70°20'
Antofagasta	23°39'	70°24'
Caldera	27°04'	70°50'

As a complementary data source, it was used data from an pelagic evaluation cruise along Peruvian coast realized by IMARPE (Instituto del Mar de Perú) in 1992. Temperature and salinity profiles data were used to compute density, Brunt-Vaissala frequency and coastal trapped waves properties. Batimetry data were taken from the NODC ETOPO-5 data set. For classification of El Niño years, we used the analyses produce by NCEP/Climate Prediction Center.

## 2.1 Wavelet Transform

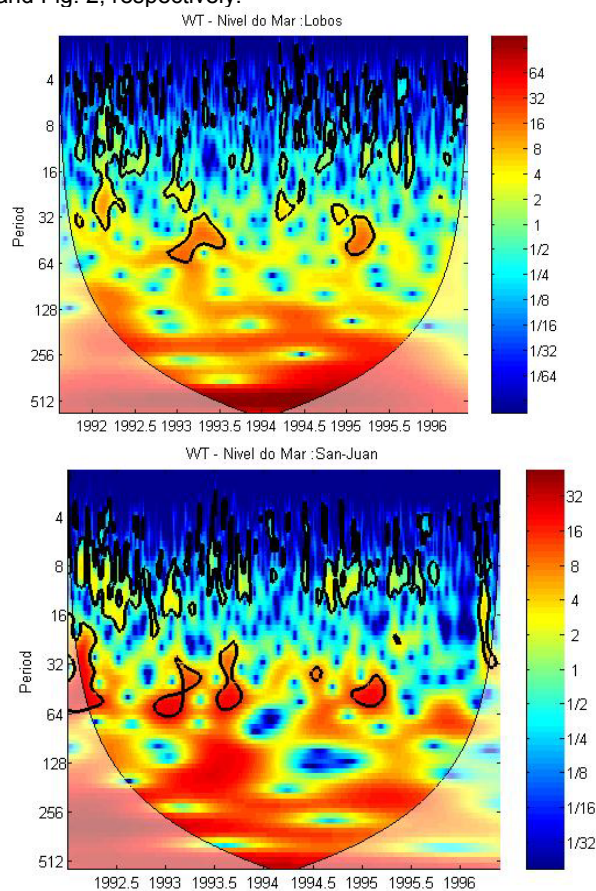
It's well known that with Fourier Transforms a limited amount of information can be retrieved about the characteristics of a time varying signal. As they inform frequencies given in the whole time series, information about an individual event would be lost (Kantha and Clayson, 2000). On the contrary, Wavelet Transform (WT) expand time series into time frequency space and can therefore find localized intermittent periodicities (Grinsted et. al., 2004). However, it must kept in mind that WT loses frequency information in the high frequency band and time information on the low-frequency band (Wang and Wang, 1996).

Wavelet analysis is based on the convolution of a function  $f(t)$  with a set of  $g(t)$  functions derived from translations and stretching of a "Mother Wavelet" (Meyers et. al., 1993). The Continuous Wavelet Transform (CWT) of a time series  $x_n$  with uniform time steps  $\Delta t$  is defined as the convolution of  $x_n$  with the scaled and normalized wavelet (Grinsted et. al., 2004).

In the present study, wavelet analysis was applied to sea level and alongshore wind stress component data (see Fig. 3 and 4), using the following methodology proposed by Torrence and Compo (1998):

- Choice a "wavelet mother" function and scales to analyze. The choice of the mother wavelet must be done in such a way that owns tributes resembling (such as asymmetry or not, strong or low temporal variation, etc) (Sá et. al., 1998). In the present work, it was used the Morlet function.
- For each scale, a normalized wavelet transform must be constructed. It's recommendable apply "padding" to the edges of the time series. Padding is a common method used to reduce one problem known such as "spectral leakage", where the amplitude of one frequency is distributed with its neighbours (Ramírez Gutierrez, 2005).
- Determine the influence cone and significance level.

For the significant spectral peaks was considered 95% of significance of level. The results of applying the CWT to some of the sea level time series and alongshore wind stress component are shown in Fig. 1 and Fig. 2, respectively.



**Figure 1.** CWT applied to sea level time series on two of the stations (Lobos and San Juan ) listed in Table 1.

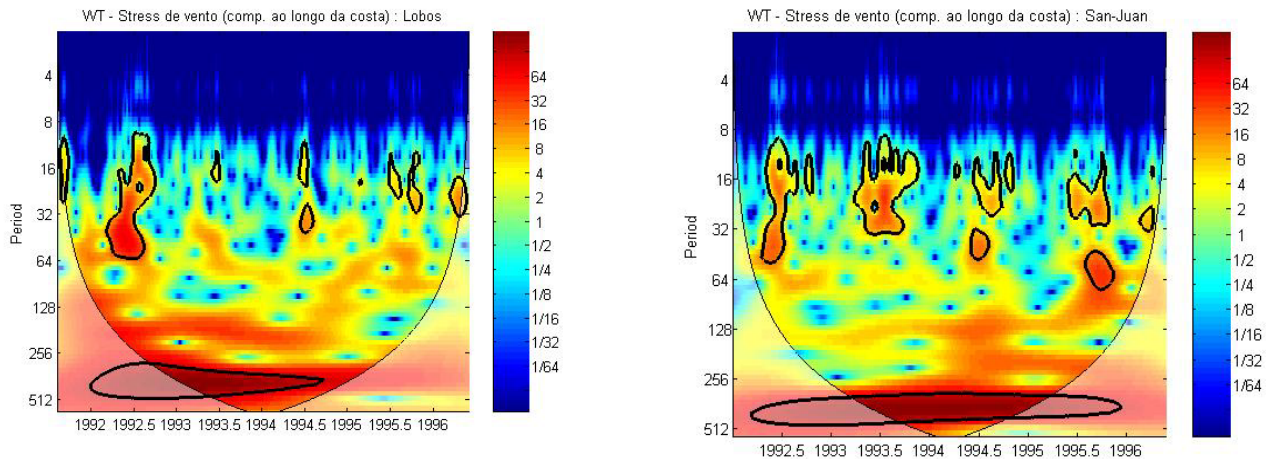


Figure 2. CWT applied to alongshore wind stress component time series on two of the stations (Lobos and San Juan) listed in Table 1

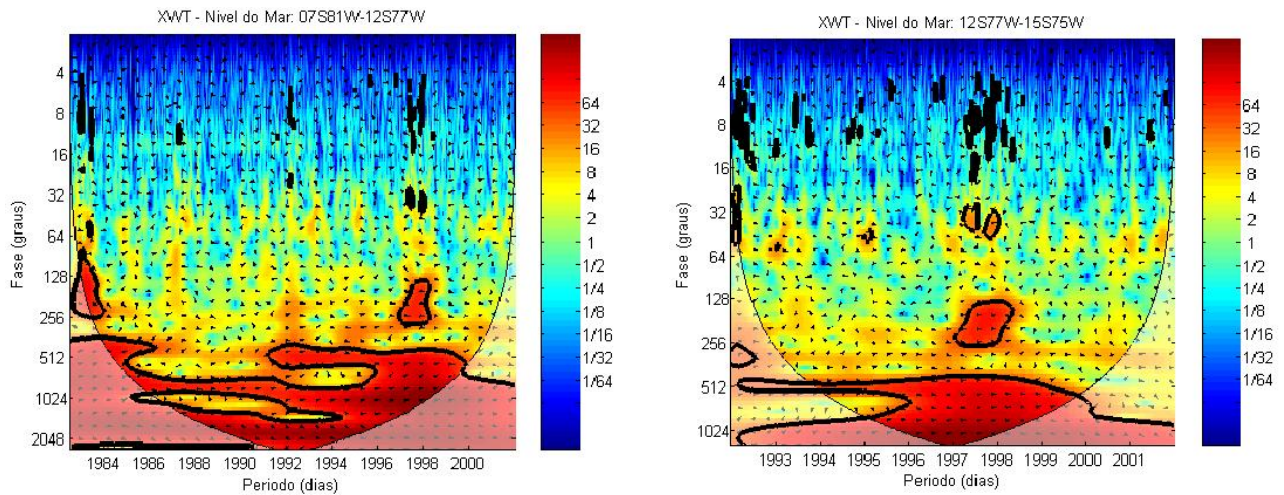


Figure 3. Results of XWT to sea level time series from two pairs of different stations.

## 2.2 The Cross Wavelet Transform (XWT)

The XWT finds regions in time-frequency space where the time series show high common power. The XWT of two series show  $X_n$  and  $Y_n$  is define as  $W^{XY} = W^X W^{Y*}$ , where  $W^X$  is the continuous wavelet transform of the time series  $X_n$  and  $*$  denotes the complex conjugate. The cross wavelet power is defined as  $|W^{XY}|$  (Grinsted et. al., 2004).

In this study, we calculated the XWT of pairs of sea level and alongshore velocity in the same location for an analysis of local forcing and to pairs of sea level time series in diferent locations to investigate the effects of remote forcing. The results of applying the XWT to some of the stations listed in table 1 are shown in Fig. 3 and Fig. 4.

## 3. Theoretical Characteristics of Coastal Trapped Waves

### 3.1 Continental shelf waves (CSW)

These waves display the following characteristics (Leblond and Mysak, 1978):

- Shelf waves are a kind of planetary topographic waves and therefore highly rotational.
- Its movement consists in a serie of horizontal eddies with alternated signal and propagates along the coast, trapped in the shelf. It progate with the coast at the left in the Southern Hemisphere.
- It's generated by big weather systems with movements along or across of the coast.
- They present long wavelengths  $L \gg \lambda$ , where  $\lambda$  it's the wide of the continental shelf and slope; low frequencies  $w \gg f$  ( $f$  is the Coriolis frequency) and small amplitudes (usually a few centimeters).
- It's believed that the generation of these waves are due to wind stress.

- It's vertical modal structure is approximately barotropic.
- According to Kantan and Clayson (2000), these waves can be well understood studying barotropic shelf waves.
- The rigid lid approach is used, because the vertical scale is smaller than the Rossby deformation.
- There exist a challenge to find analytic solutions for an arbitrary profile depth. For the simplest case:

$$\sigma = \frac{2bfk}{m^2 + k^2 + b^2},$$

$$\text{where } \tan(mL) = \frac{-m}{b+k} \quad (6)$$

and "n" has the property :

$$(n - \frac{1}{2})\pi < mL < n\pi$$

Applying this theory for the study area, we calculated the phase speed for long wavelengths (n=1, n=2 and n=3) as shown in table 2.

### 3.2 Baroclinic Kelvin waves

A kelvin wave in a two layers ocean model with constant depth  $H = H_1 + H_2$ , where  $H_1$  is the upper layer and  $H_2$  is the lower layer and a density difference among the layers of  $\Delta\rho$  has a phase speed:

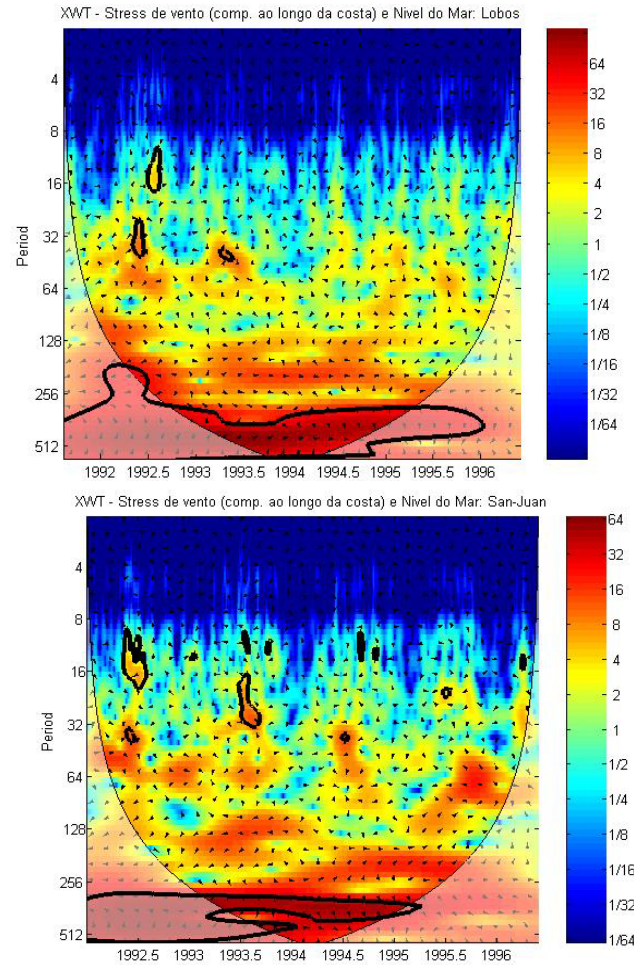
$$c = \left( \frac{\Delta\rho}{\rho} g \frac{H_1 H_2}{H_1 + H_2} \right)^{1/2} \quad (12)$$

and Rossby deformation radius is  $\delta_r = c / f$ .

Values of  $c$  and  $Rd$  for the first baroclinic mode, calculated with the observed vertical profiles of density and Brunt-Vaisala frequency ( $N^2(z)$ ) for Paita, Lobos, Callao, San Juan and Matarani stations are also shown in table 2.

**Table.2.** Theoretical phase velocities ( $c$ ) in km/day, for the barotropic continental shelf waves ( $n=1,2,3$ ) and for the first mode of baroclinic Kelvin waves. Also, it's shown the internal Rossby radius of deformation ( $Rd$ ) for  $n=1$  in km. (It was used a adjusted bathymetry to an exponential profile and density profile obtained from the oceanographic cruise of 1992).

Sections	Barotropic shelf wave			Internal Kelvin wave (n=1)	
	n=1	N=2	n=3	$Rd_i$	$cf_i$
Paita	27	35	40		
Lobos	50	44	33		
Callao	85	76	58	120	315
San Juan				86	289
Matarani	120	110	81	81	301



**Figure 4.** Results of XWT to sea level and alongshore wind stress time series, on Lobos and San Juan stations.

- Step shelf :

$$H(x) = d \quad 0 < x < L \quad (1)$$

$$H(x) = D \quad L < x < \infty \quad (2)$$

The relation of dispersion for one mode is given by:

$$\sigma = f(1-r)/(r + \coth KL), \text{ where } r = d/D \quad (3)$$

- Exponential shelf:

$$H(x) = d \exp(2bx), \quad 0 < x < L \quad (4)$$

$$H(x) = d \exp(2bL) = D \quad L < x < \infty \quad (5)$$

So, an infinite set of discrete modes exist, where the relation of dispersion is:

## 4. Results

### 4.1 Dominant periods during El Niño years

From the wavelet spectra of sea level data for coastal stations we list in Table 3 the periods in which significant peaks occurred for el Niño years. It is evident the presence of intraseasonal oscillations with periods ranging from about 20 to 90 days, in practically the entire region.

### 4.2 Local forcing

**Table 3.** Significant periods (days) in the wavelet spectra for sea level time series in some locations off Peruvian and northern Chilean coasts.

Stations	01/1983	01/1987	01/1992	01/1993	01/1997	06/1997	01/1998
02°S80°W	<20 60-90 180	20-25 70-90				25-40	<70 180
05°S81°W	<20 30-180				<40 60 180	25-40 180	
06°S80°W	<25 180		<30	40	<40 60		25-45
12°S77°W		<60	<60	20-60		<50 180	<50 180
15°S75°W			<8 20-60	15-18 55-60	50-60	<55 600	25-50 180
18°S70°W			20-60			25-75	
23°S70°W	<30	<30	<60		<20 40	40-50	

**Table 4 :** Summary of the remotely forced oscillations indicating the significant periods (days) and phase speed (km/day). Also shown are indications of the possible type of the trapped waves: IKW = Internal Kelvin Wave; CSW= Barotropic continental Shelf Wave.

Stations	Onset of 1992	04-05 1992	10/1997 -01/1998
06°S-12°S	Remote 6-1days:200-250 km/day Probably IKW	Remote -25-32 days: 180-220 km/day Probably IKW - 35-40 days: 340 km/day Probably IKW	Remote - 10 days : 300 km/day Probably IKW - 25-40 days : 200-300 km/day Probably IKW
12°S-15°S	Remote - 6-8 days: 160 km/day Probably IKW -20-30 days: 240-260km/day Probably IKW -30-50 days: 110-150 km/day Probably BSW	Remote - 10-20 days: 200 km/day Probably IKW -30-40 days: 100-150 km/day Probably BSW	

The XWT was applied to pairs of sea level data and to the alongshore wind stress to detect oscillations of local origin (Table 3). All along the coastal region studied, we found good correlations of periodicities ranging from 10 to 50 days. Off the northern peruvian coast, these oscillations occurred mainly during June 1992, which correspond to an El Niño year (El Niño 1991-92). At the southern coast, these oscillations were observed every year, but they were more intense during El Niño. Fluctuations with periods of 10 days could be associated with "Coastal Lows", which are atmospheric trapped waves. Shaffer et. al. (1997) found perturbations between 6-10 days in the Chilean coast, where synoptic-scale wind variability is probably associated with the "coastal lows" in the atmosphere (cf. Gill, 1982). It is likely that these atmospheric coastal lows could be the response to tropical convection related with El Niño.

#### 4.3 Remote forcing

XWT was applied to sea level data on different pairs of stations, with the aim to characterize propagating signals which are believed to be of remote origin. At the times in where common power was observed, there was computed phase speed using the phase spectra. In table 4, we list the dominant period T, in days, and the phase speed c for two pairs of stations (7°S-12°S and 12°S-15°S). At the beginning of

1992, between 6°S and 12°S, there was observed 6-11 days oscillations southward propagating with phase speed between 200 and 250 km/day. Between 12°S S and 15°S S, poleward traveling oscillations with periodicities between 20 and 30 days, and 30-50 days were observed. The correspondingly phase speed were of 240-260 and 110-150 km/day respectively. Between March and April 1997, in coastal region (6°S-12°S), it was observed oscillations with periods in bands of 25-32 and 35-40 days southward propagating with phase speeds of 180-220km/day and 340 km/day, respectively. Between October 1997 and January 1998, at the northern Peru, it was noticeable the presence of perturbations with periods of around 10 days and also between 20-40 days southward propagating with phase speed of 200 and 200-300 km/day respectively.

#### 4.4 Evidence of Coastal Trapped Waves : Comparisson between theoretical and observational data

Previous studies shown the presence of coastal trapped waves of frequency with typical periods of 5-20 days. Brink et al. (1983) associated these waves with remote wind forcing. Strub and Mesias (1998), mentioned that at periods of days to weeks, one of the major findings off Peru is that neither local winds nor those located closer to the equator are well correlated with sea level or alongshore currents

**Table 5.** Significant periods (days) for the cross wavelet (left) and calculated phase velocities in km/day (right) for sea level time series for two pairs of different stations.

Stations	01/1992	03/1992	04/1997	05/1997	10/1997
06°S-12°S		6-11: 200-250 km/day	26-32: 180-220 35-40: 340		10-12 : 300 26-38 : 200-300
12°S-15°S	06-11 : 160 20-30 : 240-260 30-50 : 110-150	25-30:-----		12-20: 200 28-40: 100-150	

bellow the shallow Ekman layer. It was also observed by several researchers (Smith, 1978; Brink, 1982; Romea and Smith, 1983), that the sea level and the alongshore current were strongly lagged correlated with phase speeds 180 and 260 km/day, characteristic of the first baroclinic mode of coastal trapped waves. Shaffer et al. (1997), observed strong fluctuations with periods around 50 days (associated to equatorial Kelvin Waves), and between 5 and 10 days, probably associated with Mixed Rossby-Gravity waves and Inertio-Gravity waves trapped at the Equator, during El Niño 1991-92.

With CWT, we determined the dominant modes of sea level data on each station between 7°S and 15°S and then used WXT to determined the occurrence of significant common peaks at given periods. With the time shift of the occurrence of these peaks, we determined the phase speed of these oscillations. Finally, we looked for the correspondence between the observational and theoretical values. A comparison of phase speed obtained with sea level cross-spectra and the theoretical values is shown in Table 2 are resumed in Table 5.

At the beginning of 1992, southward traveling disturbances that resemble internal Kelvin trapped waves probably occurred between 6°S - 12°S, with periods in bands of 6-11 days and 20-30 days, and with phase speeds of 160 and 260 km/day respectively. These agrees quite well with several works ( Brink ,1982; Brink, 1983; Smith, 1978; Romea and Smith, 1983). In the region comprised between 12°S and 15°S, disturbances probably associated with barotropic shelf waves ( $n=1$ ) were found propagated poleward with phase speed ranging 110-150 km/day and periods between 30 - 50 days. In contrast, Shaffer (1997) found oscillations of around 50 days but with velocities of 266 km/day. Future studies will be applied to study these particular oscillations (50 days).

For may 1997, as first approximation, it was used the theoretical phase speeds obtained for the 1992 cruise. In region comprised between 6°S and 12°S, were probably occurred baroclinic kelvin waves of 25-30 days, with phase speed around of 340 km/day. More to the south (12°S-15°S), were probably presented baroclinic kelvin waves with periods in the band of 10-20 days with velocities of 200 km/day; and barotropic shelf waves of 30 to 40 days with velocities between 100 and 150 km/day.

## 5. SUMMARY AND CONCLUSIONS

Wavelet transform and cross wavelet transform were applied to long sea level and alongshore wind stress time series, with the aim of

identify intraseasonal variability along of the South American coast and their remote and local forcing. Also, it was used temperature and salinity profiles from oceanographic cruise realized in 1992 (year Niño) to compute properties of the coastal trapped waves using simples theoretical models.

For Niño years (83, 87, 92, 97 and 98), we found the presence of intraseasonal oscillations with periods between 20 and 90 days along the coastal region from 02°S to 27°S. Specifically, during months when the extraordinary 97-98 El Niño reached its maximum extension and strongest intensity (beginning og 1998), in the region around 2°S, we found coastal oscillations in a broad band of periods shorter than 70 days. In the central Peruvian coast, the region between 6°S e 15°S, our computations show oscillations shorter than 50 days. Along almost all the coast, we observed strong correlation between sea level and local alongshore wind stress in the periods around of 10 and 50 days, with larger intensity during El Niño 1991-92 and to the south. These oscillations can be associated with "Coastal Lows", that are one type of atmospheric waves trapped at the western side of the Andes.

The deformation radius calculated were larger than the topographic scale (shelf and slope) off along the coast of Peru and northern Chile. These could be and indicative that the perturbations are likely to be Kelvin waves. But small values obtained for the stratification parameter ( $S$ ) could indicate to be shelf waves (This computations were not shown). Because of this controversy, we used theoretical simples models to compute phase speeds, and so to compare with velocities calculated from the wavelet spectra.

We found that in more intense periods of the 1991-92 and 1997-98 El Niños (onset of 1992 and 1998) and between 6°S and 12°S, there were oscillations probably associated with remotely forced internal Kelvin waves, with periods in the bands of 6-11 days (for only 91-92 Niño), 20-30 days (for only 97-98 Niño), and with phase speeds between 160 and 260 km/day. For the two El Niño events, in the region comprised between 12°S and 15°S, southward barotropic shelf waves probably occurred with velocities between 110-150 km/day, and with periods in the band of 30-50 days. Shaffer (1997) observed oscillations of nearly 50 days, but with velocities of 260 km/day. For 1997, our calculations also suggest that baroclinic kelvin waves occurred for the same region, with periods of 10-20 days and velocities around of 200 km/day.

For future works, we will apply the Brink model to evaluate properties of the coastal trapped waves observed in this study. Principal attention will to be the effect of coastal trapped waves on circulation and coastal upwelling of the region.

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## References

- Bleck, R., G. Halliwell, A. Wallcraft, S. Carroll, K. Kelly e K. Rushing, 2002 : Hybrid Coordinate Ocean Model (HYCOM) User's Manual.
- Brink, H. K., D., 1982, A comparison of long coastal trapped wave theory with observations off Peru, *J. Phys. Oceanogr.*, 897-913.
- Brink, H. K., D. Halpern, A. Huyer e R. L. Smith, 1983, The physical environment of the Peruvian Upwelling system, *Pro. Oceanog.*, vol. 12, 285-305.
- Brink K. H. e D. C. Chapman, 1987, Programas for computing properties of coastal-trapped waves and Wind-driven motions over the continental shelf and slope. WHO-87-24.
- Buchwald, V. T. e J. K. Adams, 1968, The propagation of the continental shelf waves, *Proc. Roy. Soc. London, Ser. A*, 305, 235-250.
- Clarke, A. J., 1983 : The reflection of equatorial waves from oceanic boundaries. *J. Phys. Oceanogr.*, 13, 11193-1207.
- Enfield, D. B., 1987 : The intraseasonal oscillation in eastern Pacific sea levels: How is it forced?, *J. Phys. Oceanogr.* 17, 1860-1876.
- Gill, A. E., 1982, Atmosphere-Ocean Dynamics, *Academic Press*.
- Grinsted, A., J. C. Moore e S. Jevrejeva, 2004, Application of the cross wavelet transform and wavelet coherence to geophysical time series, *Nonlinear Processes in Geophysics*, 11, 561-566.
- Kantha L. H e C. A. Clayson, 2000, Numerical models of oceans and oceanic processes, *International Geophysics Series*, Volume 66.
- Leblond P. H e L. A. Mysak, 1978, Waves in the ocean, *Elsevier Oceanography, Series 20*.
- Meyers, S. D., B. G. Kelly, e J. J. O'Brien, 1993: An introduction to wavelet analysis in oceanography and meteorology: With application to the dispersion of Yanai waves, *Monthly Weather Review*, vol. 121, 2858-2866.
- Romea, R. D e R. L. Smith, 1983, Further evidence for coastal trapped waves along the Peru coast, *J. Phys. Oceanogr.*, 1341-1356.
- Sa, L.D.A., S.B.M. Sambatti e G. P. Galvão, 1998: Ondeleta de Morlet aplicada ao estudo da variabilidade do nível do rio Paraguai em Ladario, MS, *Pesq. agropec. bras. Brasília*, vol. 33, 1775-1785.
- Shaffer, G., O. Pizarro, L. Djurfeldt, S. Salinas e J. Rutllant, 1997, Circulation and low-frequency variability near the Chile Coast: Remotely-forced fluctuations during the 1991-1992 El Niño, *J. Phys. Oceanogr.*, 27, 217-235.
- Smith, R. L., 1978, Poleward propagating perturbations in currents and sea levels along the Peru coast, *Journal Geophysical Research*, vol. 83, No. c2.
- Strub, P. T., J. Mesías, V. Montecino e J. Rutllant, 1998, Coastal ocean circulation off western South America, *The Sea*, volume 11, 10.
- Torrence, C. e G. P. Compo, 1998, A practical guide to wavelet analysis, *Bulletin of American Meteorological Society*, vol. 79, 61-77.