

# Stationary solutions for an external torque-free dual-spin spacecraft with an axial nutation damper in the platform

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The stationary solutions for the motion of a dual-spin spacecraft with a mass-spring-damper system in the platform under external-free forces consideration was determined. With knowledge of the angular momentum and the total energy of the system the critical points can be reduced to four stable equilibrium states. The equilibrium that more approaches to that is waited to find in orbit operation was obtained.

**Keywords:** Satellite, Dual-spin, Stability

## 1. Introduction

This work deals with nonlinear dynamics in the motion of the dual-spin spacecrafts. The study of the dynamics of the motion of a dual-spin satellite is a special case of much interest in the space engineering and technology areas. Some remote sensing and communication satellites have rotating parts that can be represented by the dual-spin scheme. This denomination is given for satellites that combine the advantages of an oriented platform and a rotor to maintain the gyroscopic rigidity.

Several researchers has focused the aspects concerning dual-spin motion stability as a function of parametric changes or as a function of the inherent dynamics nonlinearity. This last aspect allows to study other forms of nonlinear behavior such as chaotic instabilidades. A review of the main papers about these subjects can be found in Mehan and Asokanthan (1996).

In this work a nutation damper was enclosed in the oriented part for minor inertia axis stabilization purpose. The equations of the motion have been obtained from the total system kinetic energy and using the Lagrange's equations for generalized coordinates and almost-coordinates. Under no external forces and with knowledge of the angular momentum and total energy of the system the stationary solutions can be reduced to a finite number. The objective of this work is to determine a equilibrium state - among the four existing ones - that more it approaches to that is waited to find in orbit operation.

## 2. Equations of motion

The system under investigation consists of an external torque-free, dual-spin, spacecraft with an axisymmetric rotor attached to an asymmetric platform that contains an axial nutation damper (spring-mass-dashpot).

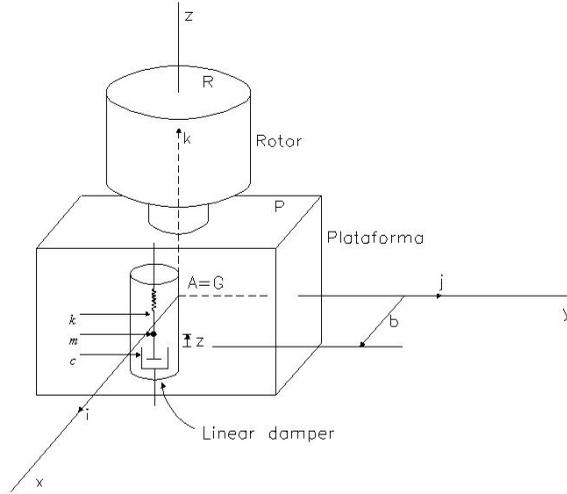


Figure 1. Dual Spinner with platform damper (Source: Kaplan (1976) and Meehan and Asokanthan (1996))

Disregarding the translational motion of the spacecraft, the degrees of freedom of the system are  $z$ ,  $\theta_x$ ,  $\theta_y$ ,  $\theta_z$  and  $\theta_r$ , where  $z$  represents the nutation damper mass position with regard the point  $A$  centered, platform fixed, reference frame;  $\theta_x$ ,  $\theta_y$ ,  $\theta_z$  represent the three axis rotations around the body fixed reference frame ( $A_{xyz}$ ) and  $\theta_r$  represents the rotation of the rotor relative to the platform. The damper is centered on the  $x$ -axis and has a point mass  $m$  that moves along a axis parallel to the  $y$ -axis at a distance  $b$  from  $A$ . The spring constant is  $k$  and the dashpot has damping constant  $c$ . The balanced rotor axis is collinear with the body fixed  $z$ -axis. The system rotates about its mass center at point  $G$  which coincides with  $A$  when  $z = 0$ . The system mass is  $m_t$  and the system principal moments of inertia are  $I_x$ ,  $I_y$ , and  $I_z$  when  $z = 0$ . The rotor has a moment of inertia  $I_r$  about the  $z$ -axis that is included in the  $I_z$ .

The system kinetic energy  $T$ , considering the system as  $n$  particles each one with mass  $m_i$ , with respect to an arbitrary reference frame with arbitrary motion (here chosen adequately as the fixed point  $A$ ) is given by

$$T = \frac{1}{2} m_t v_G^2 - \frac{1}{2} m_t v_{A/G}^2 + \frac{1}{2} \sum_{i=1}^n m_i v_{i/A}^2 \quad (2.1)$$

where  $v_G$ ,  $v_{A/G}$  and  $v_{i/A}$  respectively denote the magnitudes of  $\mathbf{v}_G$ , the velocity of the reference frame  $G_{xyz}$ ,  $\mathbf{v}_{A/G}$  the velocity of the arbitrary reference frame  $A_{xyz}$  relative to  $G_{xyz}$  and  $\mathbf{v}_{i/A}$  the velocity of the particle  $i$  with respect to  $A_{xyz}$ . The first term in Eq. 2.1 represents the kinetic energy considering total mass concentrated at  $G$ . The second term is a correction term associated with the relative motion of the reference frame  $A$  with respect to  $G$ . The third term is the kinetic energy of the system in its motion relative to the frame  $A_{xyz}$ . The second term in Eq. 2.1 can be described from displacement and velocity of  $A$  with respect to  $G$ , that is

$\mathbf{r}_{A/G} = -\mu z \mathbf{k}$ ;  $\mathbf{v}_{A/G} = -\mu z \omega_y \mathbf{i} + \mu z \omega_x \mathbf{j} - \mu \dot{z} \mathbf{k}$ ,  
where  $\mu = m/m_t$ . So

$$\frac{1}{2}m_tv_{A/G}^2 = \frac{1}{2}m\mu z^2 + \frac{1}{2}m\mu z^2\omega_y^2 + \frac{1}{2}m\mu z^2\omega_x^2 \quad (2.2)$$

The third term in Eq. 2.1 can be described as

$$\begin{aligned} \frac{1}{2}\sum_{i=1}^n m_i v_{i/A}^2 &= \frac{1}{2}(I_x + mz^2)\omega_x^2 + \frac{1}{2}(I_y + mz^2)\omega_y^2 + \frac{1}{2}I_z\omega_z^2 - mbz\omega_x\omega_z \\ &\quad + \frac{1}{2}I_y\omega_y^2 + I_y\omega_y\omega_z + \frac{1}{2}m\dot{z}^2 - mb\dot{z}\omega_y \end{aligned} \quad (2.3)$$

Substituting Eqs. 2.2 and 2.3 in Eq. 2.1 and considering  $G$  motionless, the kinetic energy of the system is

$$\begin{aligned} T &= \frac{1}{2}[I_x + m(1 - \mu)z^2]\omega_x^2 - mbz\omega_x\omega_z + \frac{1}{2}[I_y + m(1 - \mu)z^2]\omega_y^2 \\ &\quad + \frac{1}{2}m(1 - \mu)\dot{z}^2 + \frac{1}{2}I_z\omega_z^2 + \frac{1}{2}I_y\omega_y^2 + I_y\omega_y\omega_z - mb\dot{z}\omega_y \end{aligned} \quad (2.4)$$

The potential energy of the system (obtained from the spring) and the energy dissipated in the damper are

$$\begin{aligned} V &= \frac{1}{2}kz^2 \\ D &= \frac{1}{2}c\dot{z}^2 \end{aligned} \quad (2.5)$$

Lagrange's equations including non-conservative forces and viscous dissipative forces for a holonomic system with  $n$  degrees of freedom are

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial V}{\partial q_i} + \frac{\partial D}{\partial \dot{q}_i} + = Q_i \quad i = 1, 2, \dots, n \quad (2.6)$$

where  $Q_i$  represents the generalized force associated with the generalized coordinate  $q_i$ . It may also be noted that for a rotating body fixed coordinate system, Lagrange's equations become

$$\frac{d}{dt}\left[\frac{\partial T}{\partial \omega}\right] + \omega \times \left[\frac{\partial T}{\partial \omega}\right] = \mathbf{M} \quad (2.7)$$

where  $\omega = [\omega_x \ \omega_y \ \omega_z]^T$  and  $\mathbf{M} = [M_x \ M_y \ M_z]^T$ . Substitution of Eq. 2.4 in Eq. 2.7 results the Euler equations of motion for a dual spinner with platform damping,

$$\begin{aligned} I_x\dot{\omega}_x - (I_y - I_z)\omega_y\omega_z + I_r\omega_r\omega_y \\ + m(1 - \mu)\dot{\omega}_x z^2 - m(1 - \mu)\omega_y\omega_z z^2 \\ + 2m(1 - \mu)\omega_x z\dot{z} - mb\dot{\omega}_z z - mb\omega_x\omega_y z &= M_x \\ I_y\dot{\omega}_y - (I_z - I_x)\omega_x\omega_z - I_r\omega_r\omega_x \\ + m(1 - \mu)\dot{\omega}_y z^2 + m(1 - \mu)\omega_x\omega_z z^2 \\ + 2m(1 - \mu)\omega_y z\dot{z} - mb\ddot{z} + mb\omega_x^2 z - mb\omega_z^2 z &= M_y \\ I_z\dot{\omega}_z - (I_x - I_y)\omega_x\omega_y + I_r\dot{\omega}_r \\ + mb\omega_y\omega_z z - 2mb\omega_x\dot{z} - mb\dot{\omega}_x z &= M_z \end{aligned} \quad (2.8)$$

These three equations in five unknowns  $\omega_x$ ,  $\omega_y$ ,  $\omega_z$ ,  $\omega_r$  and  $z$  require two additional relationships associated with wheel torque and damper force balance. The sum of the coefficients of  $\dot{\omega}_x$  and  $\dot{\omega}_y$  are the instantaneous principle moments of inertia of the system about  $A$  in the  $x$  and  $y$  directions. The terms involving  $\dot{z}$  and  $\ddot{z}$  represents moments arising from the Coriolis and relative acceleration forces with respect to  $G$ . All other terms are combinations of gyroscopic torques.

Substituting Eqs. 2.4 and 2.5 in Eq. 2.6, yields the equation of motion for  $q_1 = \theta_r$  and  $Q_2 = T_r$ , as

$$T_r = I_r(\dot{\omega}_z + \dot{\omega}_r) \quad (2.9)$$

describing the angular acceleration-torque balance governing the relative motion of the rotor.

The acceleration-force balance of the spring mass damper system may be derived for  $q_2 = z$  and  $Q_2 = 0$ , as

$$m(1 - \mu)\ddot{z} + c\dot{z} + kz - m(1 - \mu)(\omega_x^2 + \omega_y^2)z + mb\omega_x\omega_z - mb\dot{\omega}_y = 0 \quad (2.10)$$

where the three last terms represent the centrifugal, gyroscopic and angular accelerations forces.

### 3. Stability analysis

The stability of the system was examined by considering the homogenous equations of Eqs. 2.8 to 2.10 which represents the condition of no external torque ( $\mathbf{M} = 0$ ). The critical points were then found by setting  $\dot{z} = \ddot{z} = \dot{\omega}_x = \dot{\omega}_y = \dot{\omega}_r = 0$  and  $T_r = M_x = M_y = M_z = 0$  to obtain four equilibrium states described by,

$$\begin{cases} z = \bar{z} \neq 0, \omega_x = \bar{\omega}_x, \omega_y = \pm \sqrt{\frac{(I_x - I_y - m(1-\mu)\bar{z}^2)\bar{\omega}_x^2 + k\bar{z}^2}{m(1-\mu)\bar{z}^2}}, \\ \omega_z = \frac{(I_x - I_y)\bar{\omega}_x}{mb\bar{z}}, \omega_r = \frac{[(I_y - I_z + m(1-\mu)\bar{z}^2)(I_x - I_y) + m^2 b^2 \bar{z}^2]\bar{\omega}_x}{mbI_r \bar{z}} \end{cases} \quad (3.1)$$

$$\begin{cases} z = \bar{z} \neq 0, \omega_x = \bar{\omega}_x \neq 0, \omega_y = 0, \omega_z = \frac{[m(1-\mu)\bar{\omega}_x^2 - k]\bar{z}}{mb\bar{\omega}_x} \\ \omega_r = \frac{\{[m(1-\mu)(I_x - I_z) + m^2 b^2]\bar{\omega}_x^4 - k\{[I_x - I_z - m(1-\mu)\bar{z}^2]\bar{\omega}_x^2 + k\bar{z}^2\}\}\bar{z}}{mbI_r \bar{\omega}_x^3} \end{cases} \quad (3.2)$$

$$\{z = 0, \omega_x = 0, \omega_y = 0, \omega_z = \bar{\omega}_z, \omega_r = \bar{\omega}_r\} \quad (3.3)$$

$$\{z = 0, \omega_x = 0, \omega_y = \bar{\omega}_y, \omega_z = \bar{\omega}_z, \omega_r = \frac{(I_y - I_z)\bar{\omega}_z}{I_r}\} \quad (3.4)$$

where  $\bar{\omega}_x$ ,  $\bar{\omega}_y$ ,  $\bar{\omega}_z$ ,  $\bar{\omega}_r$  and  $\bar{z}$  indicate constant values which depend on the initial conditions of the problem.

Notes:

1. The stability configurations of the system may be determined completely with knowledge of the angular momentum and initial energy of the system, reducing the equilibrium states to a finite number of possible critical points.

2. If  $\bar{\omega}_x = 0$  in Eq. 3.1

$$z = \bar{z} \neq 0, \quad \omega_y = \pm \sqrt{\frac{k}{m(1-\mu)}}, \quad \omega_z = 0, \quad \omega_r = 0$$

the motion reduces to that of a planar rigid body with nutation damper rotating about the  $y$ -axis.

3. The purpose of the present analysis is to investigate the equilibrium state described by Eq. 3.2, as it may represent a condition close to expected from a dual-spin satellite in normal operation under certain conditions.

For the purpose of local linear stability analysis the system state equations were used to obtain the characteristic Jacobian matrix and the eigenvalues of the system. These results is very cumbersome so the parametric configurations of the system had been investigated for further meaningful results. This analysis revealed that the equilibrium state (Eq. 3.2) concerned was stable for the satellite parametric configuration showed in Table 1.

#### 4. Critical points

In the absence of external torque the constant norm of the angular momentum vector is given by

$$\begin{aligned} H^2 = & [(I_x + mz^2)\omega_x - mb\omega_z z]^2 \\ & + [(I_y + mz^2)\omega_y - mb\dot{z}]^2 + [-mb\omega_x z + I_z\omega_z + I_r\omega_r]^2 \end{aligned} \quad (4.1)$$

Substituting 3.2 in 4.1 we find

$$\begin{aligned} H^2 = & [(I_x + m\bar{z}^2)\bar{\omega}_x - \frac{[m(1-\mu)\bar{\omega}_x^2 - k]\bar{z}^2}{\bar{\omega}_x}]^2 \\ & + [-mb\bar{\omega}_x\bar{z} + I_z \frac{[m(1-\mu)\bar{\omega}_x^2 - k]\bar{z}}{mb\bar{\omega}_x}] \\ & + \frac{\{[m(1-\mu)(I_x - I_z) + m^2 b^2]\bar{\omega}_x^4 - k[(I_x - I_z - m(1-\mu)\bar{z}^2)\bar{\omega}_x^2 + k\bar{z}^2]\}}{mb\bar{\omega}_x^3}]^2 \end{aligned} \quad (4.2)$$

Substituting 3.2 in the total energy of the system ( $T + V + D$ ) we find

$$\begin{aligned} E = & \frac{1}{2}[I_x + m(1-\mu)\bar{z}^2]\bar{\omega}_x^2 - [m(1-\mu)\bar{\omega}_x^2]\bar{z}^2 + \frac{1}{2}I_z \left\{ \frac{[m(1-\mu)\bar{\omega}_x^2 - k]\bar{z}}{mb\bar{\omega}_x} \right\}^2 \\ & + \frac{1}{2}I_r \left( \frac{\{[m(1-\mu)(I_x - I_z) + m^2 b^2]\bar{\omega}_x^4 - k\{[I_x - I_z - m(1-\mu)\bar{z}^2]\bar{\omega}_x^2 + k\bar{z}^2\}}{mbI_r\bar{\omega}_x^3} \right)^2 \\ & + I_r \frac{[m(1-\mu)\bar{\omega}_x^2 - k]\bar{z}}{mb\bar{\omega}_x} \left( \frac{\{[m(1-\mu)(I_x - I_z) + B^2]\bar{\omega}_x^4 - k\{[I_x - I_z - m(1-\mu)\bar{z}^2]\bar{\omega}_x^2 + k\bar{z}^2\}}{mbI_r\bar{\omega}_x^3} \right) \bar{z} \\ & + \frac{1}{2}k\bar{z}^2 \end{aligned} \quad (4.3)$$

So, given  $H$  and  $E$ , the Eqs. 4.2 and 4.3 represent a two equations system with two unknowns which can be solved for  $\bar{z}$  and  $\bar{\omega}_x$ .

Table 1. *Satellite parametric configuration adopted (After Meehan and Asokanathan, 1996)*

parameter	value
$m$	= 4.0 kg
$\mu$	= 0.00554
$b$	= 1.0 m
$I_x$	= 505.708 kgm <sup>2</sup>
$I_y$	= 466.390 kgm <sup>2</sup>
$I_z$	= 471.814 kgm <sup>2</sup>
$I_r$	= 330.812 kgm <sup>2</sup>
$k$	= 8.7 N/m
$c$	= 0.4 Ns/m
$H$	= 1905.7 Nms

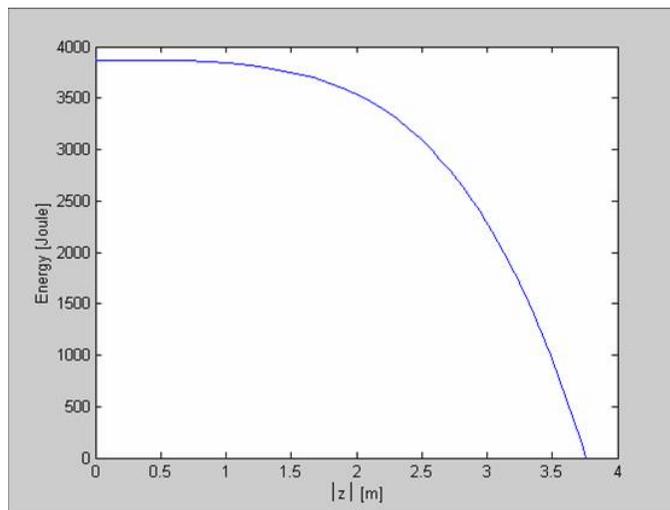


Figure 2. System energy versus absolute value of the damper displacement.

## 5. Parametric configuration

The satellite parametric configuration was adopted similar to those expected for a communication satellite. The parameters values are shown in Table 1.

## 6. Results

The following figures show the plots of the stable equilibrium state (Eq. 3.2) for the satellite configuration given in the Table 1.

## 7. Conclusions

The Fig. 2 shows that for a given total system energy there are at least *two stable equilibrium points* for  $z$  equidistance about the zero damper mass position. The Figs. 3, 4 and 5 show that there are at least *four stable equilibrium points* for  $\omega_x$ ,  $\omega_z$  and  $\omega_r$  angular velocities equidistance about the zero damper mass position ( $\omega_y = 0$ ). As

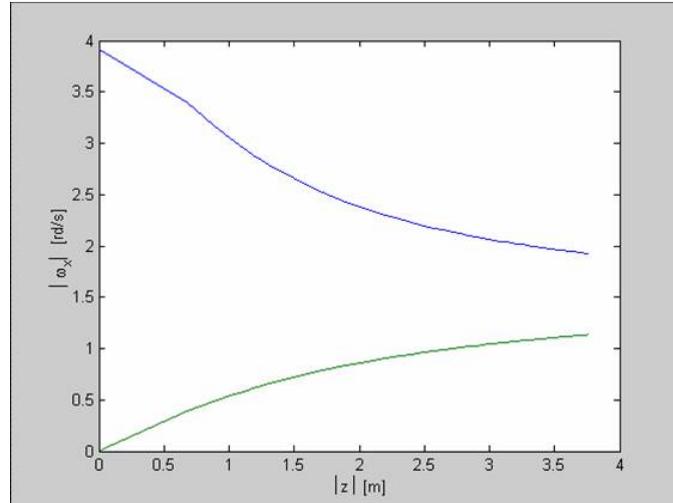


Figure 3. Absolute value of  $\omega_x$  angular velocity versus absolute value of the damper displacement.

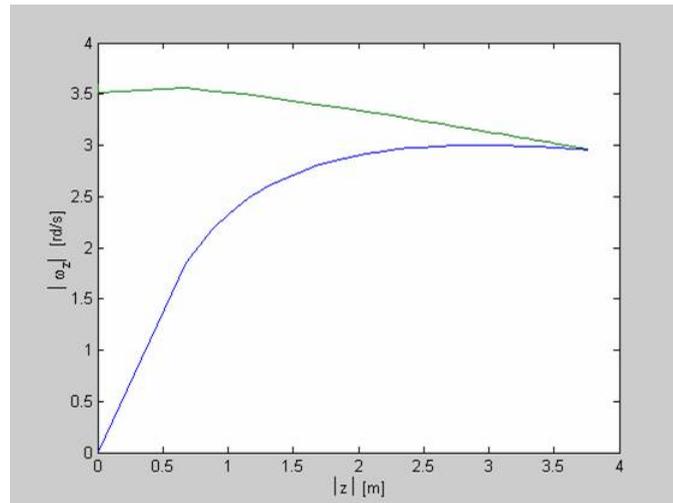


Figure 4. Absolute value of  $\omega_z$  angular velocity versus absolute value of the damper displacement.

has been found with many systems of this kind if a sinusoidally forcing rotor torque is applied the system can to jump chaotically between the stable equilibrium points. Such situations occur in practice under control system malfunction or during spinup of an unbalanced rotor or due to motor drive voltage fluctuations. Meehan and Asokanthan have shown that the system exhibit chaotic motion for a range of rotor torque perturbation amplitude and frequency. For increasing torque amplitudes the onset for chaotic motion was characterized by periodic doubling as well as Hopf bifurcations.

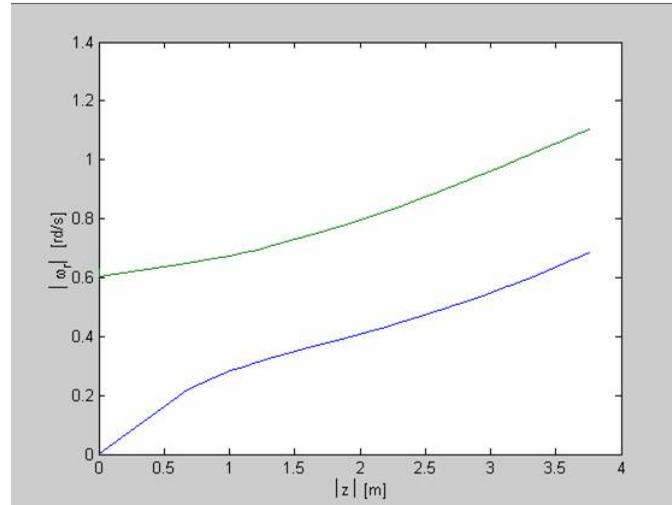


Figure 5. Absolute value of  $\omega_r$ , angular velocity versus absolute value of the damper displacement.

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