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COSMOLOGICAL CONSTRAINTS FROM HUBBLE PARAMETER ON $F(R)$ COSMOLOGIES

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ABSTRACT

Modified $f(R)$ gravity in the Palatini approach has been presently applied to Cosmology as a realistic alternative to dark energy. In this concern, a number of authors have searched for observational constraints on several $f(R)$ gravity functional forms using mainly data of type Ia supenovae (SNe Ia), Cosmic Microwave Background (CMB) radiation and Large Scale Structure (LSS). In this paper, by considering a homogeneous and isotropic flat universe, we use determinations of the Hubble function $H(z)$, which are based on differential age method, to place bounds on the free parameters of the $f(R) = R - \beta/R^n$ functional form. We also combine the $H(z)$ data with constraints from Baryon Acoustic Oscillations (BAO) and CMB measurements, obtaining ranges of values for n and β in agreement with other independent analyses. We find that, for some intervals of n and β, models based on $f(R) = R - \beta/R^n$ gravity in the Palatini approach, unlike the metric formalism, can produce the sequence of radiation-dominated, matter-dominated, and accelerating periods without need of dark energy.

Subject headings: Cosmology: cosmological parameters — Cosmology: observations

1. INTRODUCTION

Nowadays, one of the key problems at the interface between fundamental physics and cosmology is to understand the physical mechanism behind the late-time acceleration of the Universe. In principle, this phenomenon may be the result of unknown physical processes involving either modifications of gravitation theory or the existence of new fields in high energy physics. Although the latter route is most commonly used, which gives rise to the idea of a dark energy component (see, e.g., (Peebles & Ratra 2003; Padmanabhan 2003; Copeland *et al.* 2006; Alcaniz 2006; Dev *et al.* 2003)), following the former, at least two other attractive approaches to this problem can be explored. The first one is related to the possible existence of extra dimensions, an idea that links cosmic acceleration with the hierarchy problem in high energy physics, and gives rise to the so-called brane-world cosmology (Randall & Sundrum 1999; Deffayet *et al.* 2002; Alcaniz 2002; Sahni & Shtanov 2003; Maia *et al.* 2005). The second one, known as $f(R)$ gravity, examine the possibility of modifying Einstein's general relativity (GR) by adding terms proportional to powers of the Ricci scalar R to the Einstein-Hilbert Lagrangian (Kerner 1982; Barrow & Ottewill 1983; Barrow & Cotsakis 1988; Li & Barrow 2007).

The cosmological interest in $f(R)$ gravity dates back at least to the early 1980s and arose initially from the fact that these theories may exhibit an early phase of accelerating expansion without introducing new degrees of freedom (Starobinsky 1980). Recently, $f(R)$ gravity began to be thought of as an alternative to dark energy (Capozziello *et al.* 2005; Carroll *et al.* 2004; Soussa & Woodard 2004; Nojiri & Odintsov 2004) and a number of authors have explored their theeoretical and observational consequences also in this latter context. As a consequence, many questions have been raised and there is nowadays a debate about the viability of such theories (see, e.g., (Amendola *et al.* 2007a; Amendola *et al.* 2007b; Capozziello *et al.* 2006)). However, it seems that most of problems pointed out cannot be generalized for all functional forms of $f(R)$. For example, it has been shown that specific forms of the function $f(R)$ may be consistent with both cosmological and solar system-tests (Hu & Sawicki 2007; Starobinsky 2007). By considering non-minimum coupling between $f(R)$ and the Lagrangian density of matter, (Bertolami *et al.* 2007; Bertolami & Páramos 2005) discussed connections with MOND theory as well as comparison with solar observables. Besides, by starting from general principles such as the so-called energy conditions, and by generalizing them to $f(R)$ gravity, (Santos *et al.* 2007) have shown how to place broad constraints to any class of $f(R)$ theory.

Another important aspect worth emphasizing concerns the two different variational approaches that may be followed when one works with modified gravity theories, namely, the metric and the Palatini formalisms (see, e.g., (Sotiriou & Liberati 2007)). In the metric formalism the connections are assumed to be the Christoffel symbols and variation of the action is taken with respect to the metric, whereas in the Palatini variational approach the metric and the affine connections are treated as independent fields and the variation is taken with respect to both. In fact, these approaches are equivalents only in the context of GR, that is, in the case of linear Hilbert action; for a general $f(R)$ term in the action they give different equations of motion.

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For the metric approach, a great difficulty in pratice is that the resulting field equations are fourth order coupled differential equations which presents quite unpleasant behavior. In addition, simplest $f(R)$ gravity models of the type $f(R) = R - \beta/R^n$ have shown difficulties in issues such as passing the solar system tests (Chiba 2003; Amendola & Tsujikawa 2008), having the correct Newtonian limit (Sotiriou 2006a; Sotiriou 2006b) and gravitational stability (Dolgov & Kawasaki 2003). In a recent study (Amendola *et al.* 2007a; Amendola *et al.* 2007b) have shown that these theories cannot produce a standard matter-dominated era followed by an accelerated expansion.

On the other hand, the Palatini variational approach provides second order differential field equations which can also account for the present cosmic acceleration without need of dark energy. Recent studies (Amarzguioui *et al.* 2006; Fay *et al.* 2007) have shown that the above cited power-law functional forms are capable of producing the last three phases of the cosmological evolution, i.e., radiation-dominated, matter-dominated, and late time accelerating phases. Some issues still of debate in literature are whether $f(R)$ theories in Palatini formalism satisfy the solar system tests and have the correct Newtonian limit (Nojiri & Odintsov 2003; Olmo 2007; Faraoni 2006a) and whether they are free of gravitational instabilities (Faraoni 2006b; Meng & Wang 2004).

From the observational viewpoint, however, it is important to look into whether these theories of gravity are indeed compatible with different kinds of currently available cosmological data. In particular, the observational viability of some functional forms of $f(R)$ gravity have been studied using mainly data of SNe Ia and CMB radiation (Amarzguioui *et al.* 2006; Fairbairn & Rydbeck 2007; Fay *et al.* 2007).

In this paper, by following (Samushia & Ratra 2006), we use determinations of the Hubble parameter as a function of redshift (Jimenez & Loeb 2002) to derive constraints on the parameters of the $f(R) = R - \beta/R^n$ theory of gravity in the Palatini approach. These determinations, based on differential age method, relates the Hubble parameter $H(z)$ directly to measurable quantity dt/dz and can be achieved from the recently released sample of old passive galaxies from Gemini Deep Deep Survey (GDDS) (Abraham *et al.* 2004; McCarthy *et al.* 2004) and archival data (Dunlop *et al.* 1996; Spinrad *et al.* 1997; Nolan *et al.* 2001). The same data, along with other age estimates of high-z objects, were recently used to reconstruct the shape and redshift evolution of the dark energy potential (Simon *et al.* 2005), to place bounds on holography-inspired dark energy scenarios (Yi & Zhang 2007), as well as to impose constraints on the dark energy equation of state parameter (w) by transforming the selected GDDS observations into look-back time determinations (Dantas *et al.* 2007). We also combine $H(z)$ data with BAO (Eisenstein *et al.* 2005) and the CMB shift parameters (Spergel *et al.* 2007) to better constrain the free parameters of our $f(R)$ model. A brief discussion on the cosmic eras in the context of the Palatini approach is also included.

2. BASIC EQUATIONS IN THE PALATINI APPROACH

The simplest action that defines an $f(R)$ gravity is given by

$$S = \frac{1}{2\kappa}\int d^4x\sqrt{-g}f(R) + S_m\,, \tag{1}$$

where $\kappa = 8\pi G$, G is the gravitational constant and S_m is the standard action for the matter fields. Here $R = g^{\alpha\beta}R_{\alpha\beta}(\tilde{\Gamma}^{\rho}_{\mu\nu})$ and $\tilde{\Gamma}^{\rho}_{\mu\nu}$ is the affine connection, which in the Palatini approch is different from the Levi-Civita connection $\Gamma^{\rho}_{\mu\nu}$.

By varying the action with respect to the metric components we obtain the field equations

$$f_R R_{\mu\nu}(\tilde{\Gamma}) - \frac{f}{2}g_{\mu\nu} = \kappa T_{\mu\nu}\,, \tag{2}$$

where $f_R = df/dR$ and $T_{\mu\nu}$ is the matter energy-momentum tensor which, for a perfect-fluid, is given by $T_{\mu\nu} = (\rho_m + p_m)u_\mu u_\nu + p_m g_{\mu\nu}$, where ρ_m is the energy density, p_m is the fluid pressure and u_μ is the fluid four-velocity. Variation of action (1) with respect to the connection provides the equation that determines the generalized connection: $\tilde{\nabla}_\beta[f_R\sqrt{-g}g^{\mu\nu}] = 0$, where $\tilde{\nabla}$ is the covariant derivative with respect to the affine connection $\tilde{\Gamma}^{\rho}_{\mu\nu}$. This equation implies that one can write the conection $\tilde{\Gamma}$ as the Levi-Civita connection of a conformal metric $\gamma_{\mu\nu} = f_R g_{\mu\nu}$ (Li *et al.* 2007b; Koivisto & Kurki-Suonio 2006). The generalized Ricci tensor is written in terms of this connection as

$$R_{\mu\nu}(\tilde{\Gamma}) = \tilde{\Gamma}^{\alpha}_{\mu\nu,\alpha} - \tilde{\Gamma}^{\alpha}_{\mu\alpha,\nu} + \tilde{\Gamma}^{\alpha}_{\alpha\lambda}\tilde{\Gamma}^{\lambda}_{\mu\nu} - \tilde{\Gamma}^{\alpha}_{\mu\lambda}\tilde{\Gamma}^{\lambda}_{\alpha\nu}\,. \tag{3}$$

We next consider an homogeneous and isotropic universe and investigate the cosmological dynamics of $f(R)$ gravity in a flat Friedmann-Lemaitre-Roberston-Walker (FLRW) background metric $g_{\mu\nu} = diag(-1, a^2, a^2, a^2)$, where $a(t)$ is the cosmological scale factor. By expressing the generalized Ricci tensor (3) in terms of the Ricci tensor $R_{\mu\nu}(g)$ associated with the metric $g_{\mu\nu}$ we obtain the generalized Friedmann equation (Vollick 2003)

$$6f_R\left(H + \frac{f_{RR}\dot{R}}{2f_R}\right)^2 - f = \kappa\rho_m \tag{4}$$

where $H = \dot{a}/a$ is the Hubble parameter and a dot denotes derivative with respect to the cosmic time t. Here, we adopt the notation $f_R = df/dR$, $f_{RR} = d^2f/dR^2$ and so on. The trace of Eq. (2) gives

$$f_R R - 2f = -\kappa\rho_m\,, \tag{5}$$

where we have considered the fluid as a pressureless dust satisfying the conservation equation $\dot{\rho}_m + 3H\rho_m = 0$. By combining this equation with the time derivative of Eq. (5) we find

$$\dot{R} = \frac{3\kappa H\rho_m}{f_{RR}R - f_R}\,. \tag{6}$$

Now, by substituing Eq. (6) into Eq. (4) we obtain

$$H^2 = \frac{2\kappa\rho_m + f_R R - f}{6f_R\xi^2}\,, \tag{7}$$

where

$$\xi = 1 - \frac{3}{2}\frac{f_{RR}(f_R R - 2f)}{f_R(f_{RR}R - f_R)}\,. \tag{8}$$

Note that the usual Friedmann equations are fully recovered from the above expressions if $f(R) = R$, in which case the action (1) reduces to the Einstein-Hilbert one.

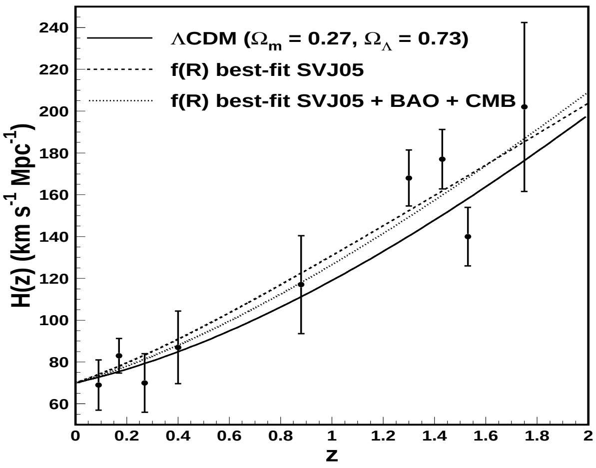

FIG. 1.— The Hubble parameter $H(z)$ as a function of the redshift for the best-fit values of n and Ω_{mo} using $H(z)$ data only and a combined fit including BAO and CMB shift measurements. For the sake of comparison, the standard ΛCDM model prediction is also shown. The data points are the measurements of the $H(z)$ by (Simon *et al.* 2005).

2.1. *Parameterization*

In this work we are particularly interested in testing the viability of a general functional form given by

$$f(R) = R - \beta/R^n \,. \tag{9}$$

In a recent paper, (Amarzguioui *et al.* 2006) have found that this model can be compatible with the supernova "Gold" data set from (Riess *et al.* 2004) for a given interval of the parameters β and n. More recently, (Fay *et al.* 2007) have shown that models of this kind are compatible with the Supernova Legacy Survey (SNLS) data (Astier *et al.* 2005) and also found narrow ranges for the values of n and β consistent with that from (Amarzguioui *et al.* 2006). Here we will follow the numerical scheme used by (Fay *et al.* 2007) to obtain the Hubble function $H(z)$.

Firstly, we rewrite Eqs.(5) and (7) in terms of redshit parameter $z = a_0/a - 1$ and the density $\rho_m = \rho_{mo}(1+z)^3$:

$$f_R R - 2f = -3H_0^2\Omega_{mo}(1+z)^3 \,, \tag{10}$$

and

$$\frac{H^2}{H_0^2} = \frac{3\Omega_{mo}(1+z)^3 + f/H_0^2}{6 f_R \xi^2} \tag{11}$$

with

$$\xi = 1 + \frac{9}{2}\frac{f_{RR}}{f_R}\frac{H_0^2\Omega_{mo}(1+z)^3}{Rf_{RR} - f_R} \,. \tag{12}$$

where $\Omega_{mo} \equiv \kappa\rho_{mo}/(3H_0^2)$. An important aspect worth emphasizing at this point is that Eqs. (10) and (11) evaluated at $z = 0$ impose a relation among n, Ω_{mo} and β, so that specifying the values of two of these parameters the third is automatically fixed. In other words, in the Palatini approach, a $f(R) = R - \beta/R^n$ theory introduces only one new parameter: n or β. In the following, we will always work with n as the free parameter.

3. ANALYSES AND DISCUSSION

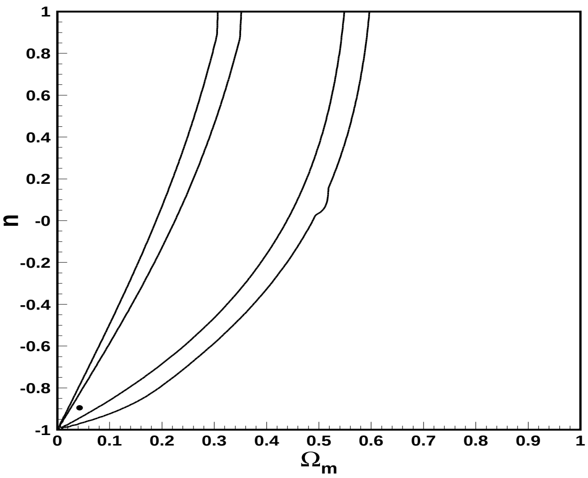

FIG. 2.— Contour plots at 95.4% and 99.73% c.l. in the $n \times \Omega_{mo}$ plane for a $f(R) = R - \beta/R^n$ theory using the SVJ05 sample of $H(z)$ measurements.

In order to impose constraints on models of $f(R)$ gravity given by Eq. (9), we minimize the χ^2 function

$$\chi^2 = \sum_{i=1}^{9} \frac{[H_{th}(z_i|\mathbf{s}) - H_{obs}(z_i)]^2}{\sigma^2(z_i)} \tag{13}$$

where $H_{th}(z_i|\mathbf{s})$ is the theoretical Hubble parameter at redshift z_i given by (11) which depends on the complete set of parameters $\mathbf{s} \equiv (H_0, \Omega_{mo}, n)$; $H_{obs}(z_i)$ are the values of the Hubble parameter obtained from the data selected by (Simon *et al.* 2005) (SVJ05) and $\sigma(z_i)$ is the uncertainty for each of the nine determinations of $H(z)$. In what follows, we are going to work with n and Ω_{mo} as free parameters and study the bounds on them imposed by the SVJ05 $H(z)$ data sample. The value for the Hubble parameter today is taken as $H_0 \simeq 70$ km/s/Mpc, in agreement with current estimates (Freedman *et al.* 2001).

In Figure 1 we show the evolution of the Hubble parameter with redshift for the two best-fit values for n and Ω_{mo} discussed in this paper, as well as the prediction from the standard ΛCDM model ($\Omega_{mo} = 0.27$). The three curves are superimposed on the data points of the SVJ05 sample. Note that all models seem to be able to reproduce fairly well the $H(z)$ measurements.

Figure 2 shows the first results of our statistical analyses. Contour plots (95.4% and 99.7% c.l.) in the $n \times \Omega_{mo}$ plane are shown for the χ^2 given by Eq. (13). We clearly see that the measurements of $H(z)$ alone do not tightly constrain the values of n and Ω_{mo}, allowing for a large interval of values for these parameters, with n ranging from -1 to even beyond 1, and Ω_{mo} consistent with both vacuum solutions ($\Omega_{mo} = 0$), as well with universes with up to 60% of its energy density in the form of nonrelativistic matter. The best-fit values for this analysis are $\Omega_{mo} = 0.04$, $n = -0.89$ and $\beta = 1.11$, with a reduced $\chi^2_\nu \simeq 1.17$.

Figure 3a shows the effective equation of state

$$w_{eff} = -1 + \frac{2(1+z)}{3H}\frac{dH}{dz} \tag{14}$$

as a function of the redshift for the best-fit values above. To plot this curve we have included a component of ra-

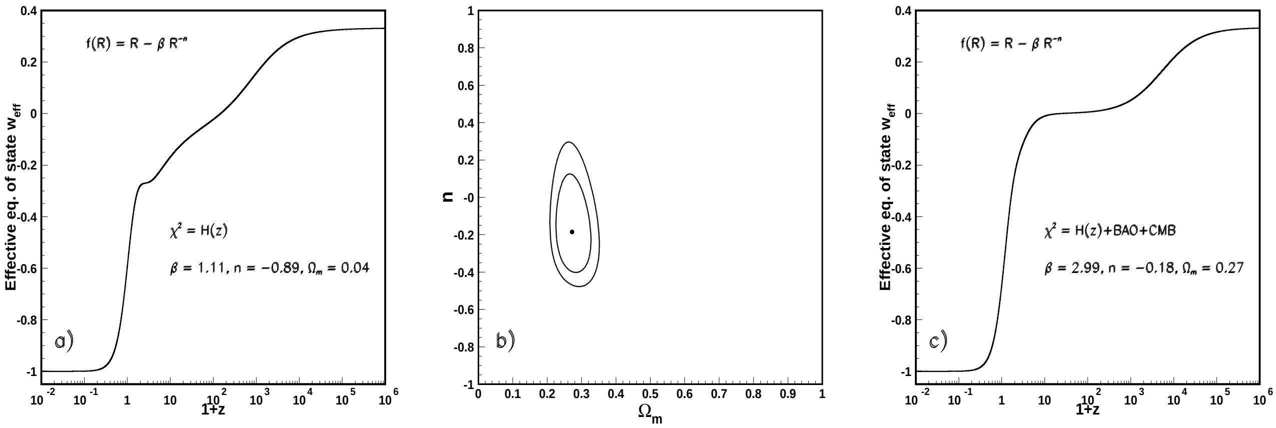

FIG. 3.— **a)** Effective equation of state [Eq. (14)] as a function of redshift for the best-fit value of n and Ω_{mo} from $H(z)$ data analysis. A radiation component with $\Omega_{\gamma o} = 5 \times 10^{-5}$ has been included. **b)** Same as in Fig. 2 when BAO and CMB shift parameters are included in the χ^2 analysis. **c)** Same as in Panel 3a when BAO and CMB shift parameters are included in the statistical analysis.

TABLE 1
BEST-FIT VALUES FOR n AND β

Test	Ref.	n	β^a
SNe Ia (SNLS)	(Fay *et al.* 2007)	0.6	12.5
SNe Ia (SNLS) + BAO + CMB	(Fay *et al.* 2007)	0.027	4.63
SNe Ia (Gold)	(Amarzguioui *et al.* 2006)	0.51	10
SNe Ia (Gold) + BAO + CMB	(Amarzguioui *et al.* 2006)	-0.09	3.6
H(z)	This Paper	-0.89	1.11
H(z) + BAO + CMB	This Paper	-0.18	3.0

[a]The ΛCDM model corresponds to $n = 0.0$ and $\beta = 4.38$.

diation $\Omega_{\gamma o} = 5 \times 10^{-5}$. It is worth mentioning that the best-fit point is not representative from the statistical point of view, given the weak power of constraining shown in Figure 2. Note also that, similarly to some results in the metric formalism (Amendola *et al.* 2007a; Amendola *et al.* 2007b), for these specific values of the n and Ω_{mo} parameters, there is no matter-dominated era followed by an accelerated expansion.

3.1. *Joint Analysis*

In (Fay *et al.* 2007) it was shown that when the measurements of SNe Ia luminosity distances are combined with information concerning the Baryon Acoustic Oscillation (BAO) peak (measured from the correlation function of luminous red galaxies) and the CMB shift parameter (which relates the angular diameter distance to the last scattering surface with the angular scale of the first acoustic peak in the CMB power spectrum), the constraining power of the fit to $f(R)$ paramenters is greatly improved. Following such an approach we examine here the effects of summing up the contributions of these two parameters into the χ^2 of Eq. (13).

In fact, when the BAO (Eisenstein *et al.* 2005)

$$A_{0.35} = \left[\left(\int_0^{0.35} \frac{dz}{H}\right)^2 \frac{z}{H(z)}\right]^{1/3} = 0.469 \pm 0.017 \quad (15)$$

and the CMB shift parameter (Spergel *et al.* 2007)[7]

$$R_{1089} = \sqrt{\Omega_{mo} H_0^2} \int_0^{1089} \frac{dz}{H} = 1.70 \pm 0.03 \quad (16)$$

are included into the fit, a considerable enhancement of the constraining power over n and Ω_{mo} takes place, as can be seen in Fig. 3b, which shows the countour curves in the $n \times \Omega_{mo}$ plane. The best-fit value ($n = -0.18$, $\beta = 3.0$, $\Omega_{mo} = 0.27$ with $\chi^2/\text{ndof} = 1.02$) is consistent with current estimates of the contribution of non-relativistic matter to the total energy density in a flat universe. The fit also constrains the parameters n to lie in the intervals $n \in [-0.5, 0.3]$ and $\beta \in [1.6, 4.4]$ at 99.7% c.l., which is consistent with the results obtained in Refs. (Amarzguioui *et al.* 2006; Fay *et al.* 2007) using the supernova *Gold* and the SNLS data sets, respectively. For the best-fit solution, the universe goes through the last three phases of cosmological evolution, i.e., radiation-dominated ($w = 1/3$), matter-dominated ($w = 0$) and the late time acceleration phase (in this case with $w \simeq -1$), as shown in Fig. 3c. In Table I we summarize the main results of this paper compare them with recent determinations of the parameters n and β from independent analyses.

4. CONCLUSIONS

[7] To include the CMB shift parameter into the analysis, the equations of motion had to be integrated up to the matter/radiation decoupling ($z \simeq 1089$), so that radiation is no longer negligible and was properly taken into account.

By considering a flat FRW cosmology we have analyzed the $f(R) = R - \beta/R^n$ theory of gravity, with equations of motion derived from the generalized action (1), according to the Palatini approach. We have performed consistency checks and tested the cosmological viability for a theory of this type by using current determinations of the Hubble parameter at different redshifts obtained from differential ages techniques. The use of these data to constrain cosmological models is interesting because, differently from distance measurements, the Hubble parameter is not integrated over. This means that the differential age method is less sensitive to systematic errors than the standard distance methods. We find that the determinations of $H(z)$, when combined with the BAO and CMB shift parameter, lead to constraints competitive to those achieved with SNe Ia *Gold* and SNLS data, as given by (Amarzguioui *et al.* 2006; Fay *et al.* 2007). The FRW cosmology correspoding to the best-fit solution for a combined $H(z)$+BAO+CMB χ^2 minimization presents all three last phases of the Universe evolution, namely, radiation era, matter era and a phase of acceleration at late times. We emphasize that even though the current data sample of $H(z)$ is small, the differential age technique is very promissing and, with the large amount of data that is expected in the near future, these observables will certainly provide strong additional constraints on cosmological parameters such as those coming from SNe Ia, CMB and LSS data.

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