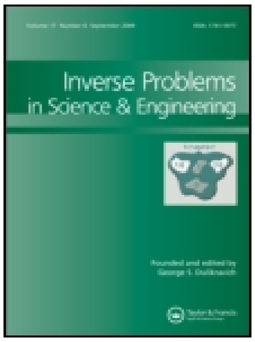

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Fuzzy ant colony optimization for estimating chlorophyll concentration profile in offshore sea water

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The determination of some inherent optical properties can be addressed by estimating the ocean chlorophyll concentration, if bio-optical models can be applied – such as for the offshore sea water. This inverse problem can be formulated as an optimization problem and iteratively solved, where the radiative transfer equation is the direct model. An objective function is given by the square difference between computed and *in situ* experimental radiances at every iteration. In the standard ant colony optimization (ACO), the pheromone is reinforced only on the best ant of the population. The fuzzy strategy consists in including additional pheromone quantity on the best ant, but a small pheromone quantity is also spread over the other solutions close to the best one. Test results show that the fuzzy-ACO produces better inverse solutions.

Keywords: ant colony optimization; fuzzy system; chlorophyll profile; radiative transfer equation; LTSN method

AMS Subject Classifications: 65K05; 90C30; 90C70; 93B40

Nomenclature

Symbol	Description
a	Absorption coefficient
a^c	Statistically derived chlorophyll specific absorption coefficient
a^w	Pure water absorption
b	Scattering coefficient
$C(z)$	Chlorophyll concentration
F	The irradiance or *net flux* of radiation integrated for all directions
L	Radiance
mit	Maximum of ants generation/iterations
na	Number of ants per generation

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na_{p}	Number of ants pre-selected
np	Number of points in the water layer
N	Number of polar angles present in the forward model
ns	Number of unknown parameters
N_{μ}	Number of points in the Gauss–Legendre quadrature
p	Phase function
S	Radiation source/sink
T_0	Initial pheromone quantity
T	Pheromone quantity
z	Depth
γ	Spreading pheromone parameter
ζ	Weight of quadrature
ρ	Pheromone decay ratio
τ	Optical thickness
η	Quadrature node
μ	Cosine of polar angle
ξ	Medium optical depth
ϖ_0	Albedo
ϕ	Azimuthal angle

1. Introduction

Optical properties in the offshore sea water can be described by means of bio-optical models, where the vertical profiles of the absorption and scattering coefficients are related with the chlorophyll profile [1]. Therefore, the determination of some inherent optical properties can be addressed by estimating the ocean chlorophyll concentration. The estimation of the chlorophyll concentration profile can be formulated as an optimization problem and iteratively solved, where the radiative transfer equation (linear Boltzmann equation) is the direct model. An objective function is given by the square difference between computed and experimental radiances.

An ant colony optimization (ACO) is a procedure based on the collective behaviour of ants moving between the nest and a food source [2]. Each ant marks its path with a certain amount of pheromone and the marked path is further employed by other ants as a reference. ACO has been used successfully to solve the travelling salesman problem (TSP) and other graph-like problems [3]. Becceneri et al. [4] transformed the function optimization problem into a graph-like problem, which was then solved using an ACO. A modified ACO was proposed by Becceneri et al. [5], in which artificial ants are allowed to deposit pheromone, not only on the edges in their paths, but also on edges close to them: the closer a given edge is to one of those in the ant's path, the more pheromone it receives. We are calling the latter strategy as *fuzzy-ACO*.

Recently, an ACO approach was used for solving inverse problems, where a pre-selection scheme of the candidate solutions is performed by their smoothness, quantified by a Tikhonov norm [6,7]. In the standard ACO method, the pheromone is only reinforced on the best ant of the population (the lowest objective function value).

In this article, the fuzzy-ACO is applied to identify chlorophyll concentration profile in offshore ocean [8]. This profile is used to compute the absorption and scattering coefficients (bio-optical models [8]). The forward problem is modelled by the integral-differential radiative transfer equation (RTE), and it is solved by using the LTS_N [9,10].

The rest of this article is organized as follows: Section 2 establishes the forward model utilized represented by the RTE. The optimization strategy proposed (ACO) as well as the algorithm employed are explained in Section 3. Section 4 provides the numerical experiments and the results obtained. The conclusions and future work are discussed in Section 5.

2. Radiative transfer equation

The forward problem is described by the RTE:

$$\mu \frac{\mathrm{d}L}{\mathrm{d}\tau} + L = \frac{\varpi_0(\tau)}{4\pi} \int_0^{2\pi} \int_{-1}^{1} p(\mu, \varphi \to \mu', \varphi') L \mathrm{d}\mu' \mathrm{d}\varphi' + S \tag{1}$$

subjected to boundary conditions

$$L_\lambda(0, \mu, \varphi) = F\delta(\mu - \mu_0)(\varphi - \varphi_0) \tag{2a}$$

and

$$L_\lambda(\tau, \mu, \varphi) = 0 \tag{2b}$$

where τ is the medium *optical depth*, $L = L(\tau, \mu, \varphi)$ is the radiance, $p(\cdot)$ is the phase function, $\varpi_0 = b/(a+b)$ is the albedo, a and b are the absorption and scattering coefficients – respectively, S is the radiation source/sink, τ is the optical thickness, $\mu \in [-1, 1]$ and $\varphi \in [0, 2\pi]$ are the cosine of polar angle ($\mu = \cos\theta$) and the azimuthal angle and λ is the wavelength. Assuming azimuthal symmetry: $L(\tau, \mu, \phi) = L(\tau, \mu)$, and $p(\mu, \phi \to \mu', \phi') = p(\mu \to \mu')$. However, here it will be considered an isotropic scattering: $p(\mu \to \mu') \equiv 1$.

The LTS_N method is employed for solving Equation (1). The discrete ordinates technique is a collocation method, where the integral term in Equation (1) is approximated by a Gauss–Legendre quadrature for an N_μ finite number of polar angles. For simplicity, all IOP are considered space-independent. Therefore, the integro-differential Equation (1) becomes a system of differential equations [10]. Expressing this system in matrix form

$$\frac{\mathrm{d}L(\tau)}{\mathrm{d}\tau} + L(\tau) = \mathbf{A}L(\tau) + S(\tau) \tag{3}$$

where the matrix entries of system (3) are as follows

$$A = \begin{cases} p_{ij}\left(\dfrac{\zeta_i}{\eta_i}\right) - \eta_i^{-1} & \text{if } i = j; \\ p_{ij}\left(\dfrac{\zeta_i}{\eta_i}\right) & \text{if } i \neq j \end{cases}$$

being ζ_is the weights of quadrature, η_i the quadrature node, that is, the i-th root of a Legendre polynomial of order N_μ, with $\boldsymbol{L}(\zeta) = [L(\tau, \eta_1) \;\; L(\tau, \eta_2) \ldots, L(\tau, \eta_{N\mu})]^{\mathrm{T}}$ and

$S(\tau) = [S(\tau, \eta_1)/\eta_1\, S(\tau, \eta_2)/\eta_2 \ldots, S(\tau, \eta_{N\mu})/\eta_{N\mu}]^{\mathrm{T}}$. The discrete phase function is represented as an expansion in Legendre polynomial $-P_l(\eta)$:

$$p_{ij} \approx \sum_{l=0}^{N_p} \beta_l P_l(\eta_i) P_l(\eta_j).$$

However, for the isotropic case: $p_{ij} \equiv 1$. Matrix problem (3) is resolved by applying the Laplace transform on space variable resulting in the following operational equation

$$(sI - A)\hat{L}(s) = L(0) + \hat{S}(s) \tag{4}$$

where $L(s) = \int_0^\infty L(\tau)e^{-s\tau}\mathrm{d}\tau$. The resolvent of Equation (4)

$$B(\tau) = L^{-1}\left\{(sI - A)^{-1}\right\} = \sum_{n=1}^{N_\mu} P^n(s_n \tau) \tag{5}$$

is obtained analytically using the Heaviside expansion technique [9], where the matrix $P = Xe^D X^{-1}$, from the spectral decomposition of the matrix $A = XDX^{-1}$. Equation (5) yields to write:

$$L(\tau) = B(\tau)L(0) + \int_0^\tau B(\tau - \varsigma)S(\varsigma)\mathrm{d}\varsigma \tag{6}$$

Finally, the vector $\boldsymbol{L}(0)$ is calculated by solving the algebraic linear system:

$$\begin{bmatrix} L^+(\zeta) \\ L^-(\zeta) \end{bmatrix} = \begin{bmatrix} B_{11}(\zeta) & B_{11}(\zeta) \\ B_{11}(\zeta) & B_{11}(\zeta) \end{bmatrix} \begin{bmatrix} L^+(0) \\ L^-(0) \end{bmatrix} + \begin{bmatrix} H_1(\zeta) \\ H_2(\zeta) \end{bmatrix} \tag{7}$$

denoting

$$H(\tau) \equiv \begin{bmatrix} H_1(\tau) & H_2(\tau) \end{bmatrix}^T = \int B(\tau - \varsigma)S(\varsigma)\mathrm{d}\varsigma. \tag{8}$$

The exponential behaviour of the solution, added to the fact that the eigenvalues s_n increase in magnitude with N_μ, implies in an adjust on the form (9). The difficulty mentioned can be avoided using a variable changing on τ:

$$B(\tau) = \sum_{n=1}^{N_\mu/2} P^n(s_n\tau) + \sum_{n=N_\mu/2+1}^{N_\mu} P^n(s_n\tau) = B^+(\tau) + B^-(\tau) \tag{9}$$

where superscripts $\pm$ denote positive and negative eigenvalues of matrix A. From this, the solution is written as

$$L(\tau) = B^-(\tau)L(0) + B^+(\tau - \zeta)L(\zeta) = H^-(\tau) + H^+(\zeta) \tag{10}$$

The convergence of the $\mathrm{LTS_N}$ method was established using the $\mathrm{C_0}$-semigroup theory [11,12].

Bio-optical models are dependent from the chlorophyll concentration in the sea, and they can be used to represent the absorption and scattering coefficients for off-shore ocean waters [8]. For the present problem, a particular chlorophyll concentration profile was considered, corresponding to the Celtic Sea:

$$C(z) = 0.2 + \frac{144}{9\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{z - 17}{9}\right)^2\right] \tag{11}$$

where z is the depth in meters and $C(z)$ is given in $\mathrm{mg\,m^{-3}}$. This profile can be seen in Section 4 of this work, termed *exact*. A bio-optical model can be applied for the absorption coefficient [8]

$$a_r(\lambda) = \left[a^w + 0.06a^c C_r^{0.65}(z)\right]\left[1 + 0.2e^{-0.014(\lambda-440)}\right] \tag{12}$$

where a^w is the pure water absorption and a^c is a nondimensional, statistically derived chlorophyll specific absorption coefficient, and λ is the considered wavelength, while another chlorophyll correlation can be formulated for the scattering coefficient [8],

$$b_r(\lambda) = \left(\frac{550}{\lambda}\right)0.30C_r^{0.62}(z). \tag{13}$$

The values of a^w and a^c depend on the wavelength and can be found in the tables provided in [8]. In this experiment, the values corresponding to a^w, a^c, λ were 0.064, 0.357 and 550 nm, respectively.

3. Fuzzy ant colony optimization

As mentioned before, the fuzzy-ACO strategy includes additional pheromone quantity over the best ant, but a small pheromone quantity is also spread on the other solutions close to the best one, decreasing the pheromone quantity as far as the solution is from the best ant (Figure 1). Each candidate solution corresponds to a discrete chlorophyll profile. The standard ACO algorithm can be described in an algorithm format.

ACO – Algorithm

begin

read parameters

first guess solution: $Q = C(0) = [1\ 1\ \ldots\ 1]^T$

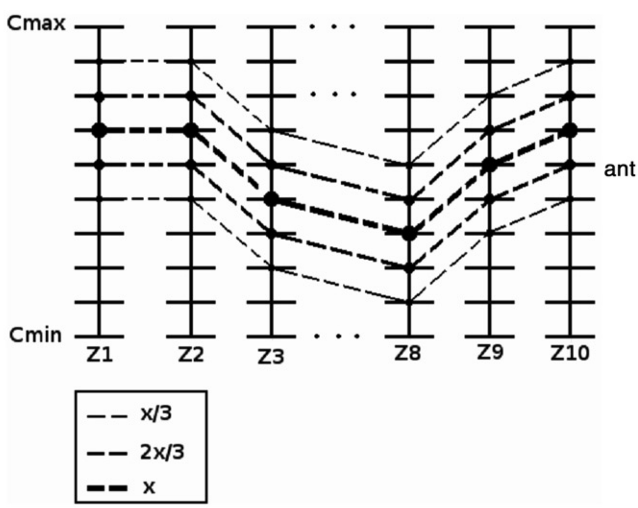

Figure 1. Pheromone quantity is spread on the three candidate solutions.

```
  compute pheromone: T_0 = 1/(ns * Q)
  initial pheromone concentration: T_ij(0) = T_0
  for ii = 1, mit do
    generation of na_p ants
    for jj = 1, na_p do
      evaluate J[C(jj)]
    end
    select the best ant: C_min
    decrease pheromone: T_ij(n+1) = (1 − ρ)T_ij(n)
    best ant (*): T_ij(n+1) = T_ij(n+1) + T_0
    compute probability: P_ij
    P_ij(n+1) = [T_ij(n+1)]^α / ∑T_ij(n+1)]^α
  end
  return C_optimum = C_min(mit)
end.
```

In the ACO method, several generations of ants are produced. For each generation, a fixed amount of ants (*na*) is evaluated. From this total amount, only a portion of ants, given by na_p is evaluated by the objective function. For instance, if the total number of ants is 90 and $na_p = 15$, from each set of 6 ants, only one – the smoothest, quantified by a Tikhonov norm – is evaluated in the objective. Taking every 6, the smoothest candidate solution is selected, until complete 90 ants, resulting only 15 calls to the objective function.

Each ant is associated to a feasible path and this path represents a candidate solution, being composed of a particular set of edges of the graph that contains all possible solutions. Each ant is generated by choosing these edges on a probabilistic basis.

This approach was successfully used for the TSP and other graph-like problems [2]. The best ant of each generation is then chosen and it is allowed to mark its path with pheromone. This will influence the creation of ants in the future generations. The pheromone are placed by the ant's decays due to an evaporation rate. Finally, at the end of all generations, the best solution is assumed to be achieved.

A solution is composed by linking *ns* nodes and in order to connect each pair of nodes, *np* discrete values can be chosen. This approach was used to deal with a continuous domain. Therefore, there are $ns \times np$ possible paths $[i, j]$ available.

For the fuzzy-ACO, the updating of pheromone for the best ant step (*) is changed to add $\gamma^j T_0$ to the path-*j*, with $\gamma < 1$. The parameters required for the ACO approach, the values utilized in the numerical experiment are shown in Table 1 below.

We cannot finish this section without commenting on the word *fuzzy* used in this paper. The idea to spread the *pheromone* over more than one path comes from the fuzzy sets (triangles). Such set shapes inspired us to formulate the new approach to the ACO. Therefore, we decide to maintain the name *fuzzy* in this approach.

4. Numerical experiments

The results are obtained considering *in situ* observational data. The number of quadrature points utilized were $N = 20$ (10 for the positive directions and 10 for the negative ones) assuming an isotropic scattering. The inversion is performed using synthetic experimental data, employing different levels of noise (0, 5 and 10%). The ocean layer is discretized with 10 sub-layers with a maximum water depth of 131.23 ft (40 m), and the ACO worked with

Table 1. ACO parameters.

Parameter	Numerical value
mit	1000
na	90
na_{p}	15
ns	10
np	10
T_0	0.1
ρ	0.03
γ^{a}	$\{0.67, 0.33\}$

Note: [a]In this case, we are considering only 2 other solutions close to the best one.

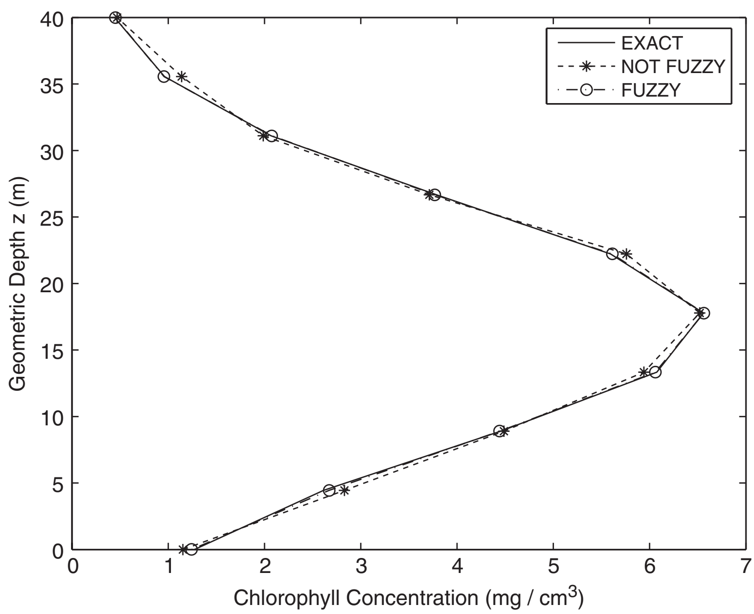

Figure 2. Chlorophyll concentration × ocean depth: exact values (solid curve), inverse solution with standard ACO (dashed curve), inverse solution with fuzzy-ACO (dash-dot curve) – noiseless data.

90 ants, and the parameter $\alpha = 1$ (ACO-algorithm). The test case worked here follows the problem treated by Souto et al. [13].

The noiseless inversion solution and the convergence plots are shown in Figures 2 and 3, respectively. For this condition, the fuzzy-ACO approach was shown to produce better results than the standard-ACO. This result is better assimilated when we look to convergence analysis made in Figure 3. Figures 4 and 5 show the inverse solution and the convergence plots, respectively, for 5% of noise level. As said before, the fuzzy-ACO improves the estimation of the chlorophyll concentration profile for the deeper part of the water layer. However, around the maximum value of the chlorophyll concentration, the inverse solution obtained with fuzzy-ACO is worse than standard ACO.

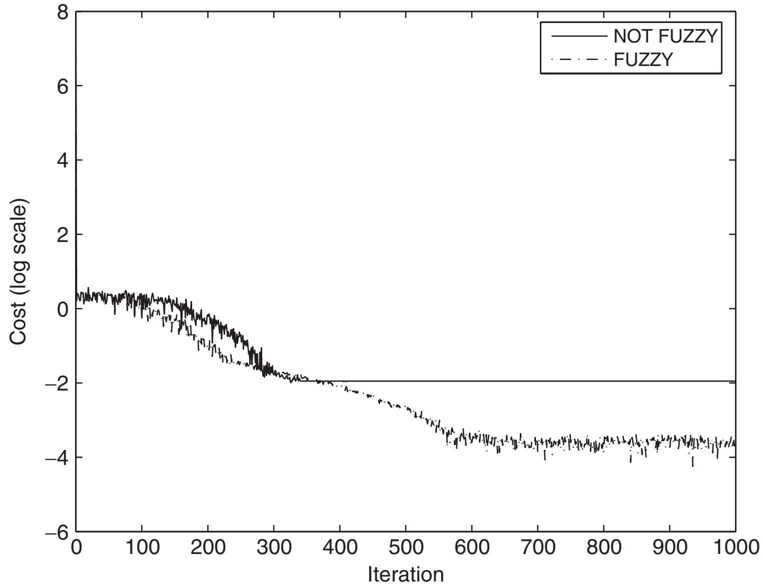

Figure 3. Convergence: fuzzy-ACO (dash-dot curve), standard-ACO (solid curve) – noiseless data.

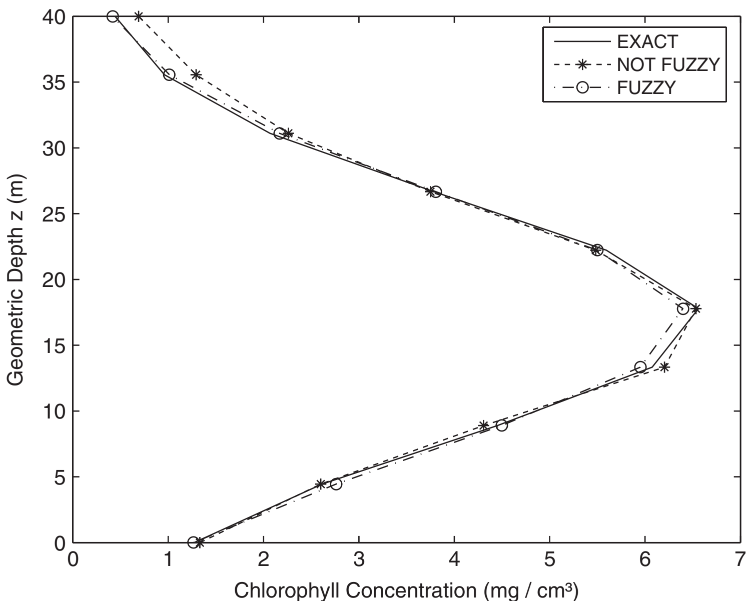

Figure 4. Chlorophyll concentration × ocean depth: exact values (solid curve), inverse solution with standard ACO (dashed curve), inverse solution with fuzzy-ACO (dash-dot curve) – 5% of noise.

Inverse solution for chlorophyll concentration with 10% of noise level is shown in Figure 6. Clearly, the fuzzy-ACO has improved the inverse solution for the deeper part of the ocean water layer. The convergence for standard-ACO and fuzzy-ACO is shown in Figure 7. From the convergence plot, it is easy to realize that the fuzzy-ACO can find

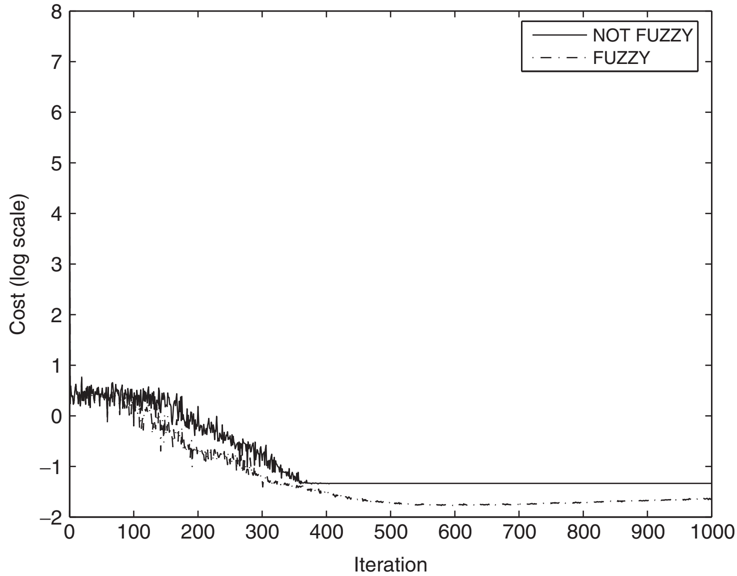

Figure 5. Convergence: fuzzy-ACO (dash-dot curve), standard-ACO (solid curve) – 5% of noise.

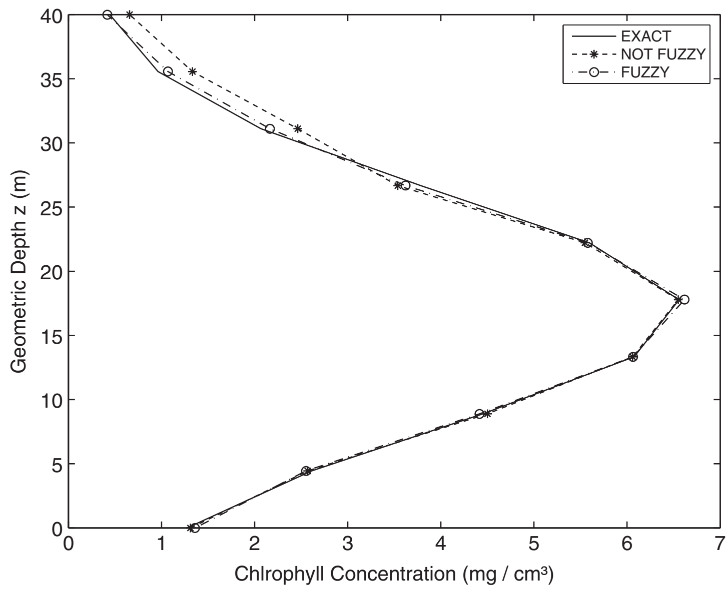

Figure 6. Chlorophyll concentration × ocean depth: exact values (solid curve), inverse solution with standard ACO (dashed curve), inverse solution with fuzzy-ACO (dash-dot curve) – 10% of noise.

better solution even for the first iterations, this behaviour is maintained for all generations – see Figure 7.

Considering the reconstructions with 5% and 10% of noise, the results applying the fuzzy-ACO suggest that the approach is effective to get better inversions for chlorophyll profile in the deeper part of the ocean water layer.

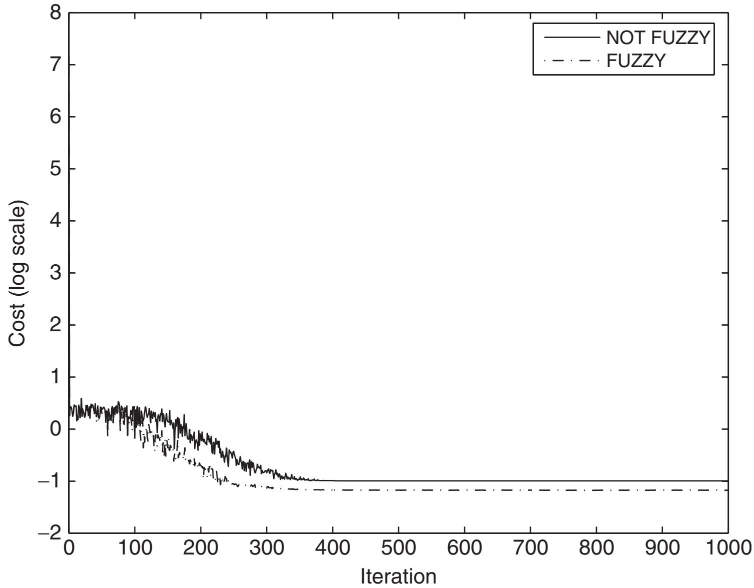

Figure 7. Convergence: fuzzy-ACO (dash-dot curve), standard-ACO (solid curve) – 10% of noise.

5. Final remarks

Inverse hydrological optics is a scientific challenge. Many research groups have dealt with several approaches trying to find out a general strategy for this subject. Here, we have introduced the fuzzy-ACO to retrieve the chlorophyll concentration profile from *in situ* radiance measurements [13] to offshore ocean water.

The fuzzy-ACO was effective to improve the inverse solution, mainly in the deeper part of the water layer. The scheme works well, even for high level of noise in the experimental data.

The results encourage us to go ahead to test this approach to other problems: anisotropic medium (see the LTS_N formulation), experiments without azimuthal symmetry, and multispectral inversion [14].

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