Multiscale analysis of Eta forecasts: Preliminary analysis

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Motivation of this work

Are the short and long range Eta model runs seeing the same time scales?
Wavelet analysis

Tool to understand the \textit{multiscale} aspects of functions or signals.

\textbf{Synthesis and synergy} of:

- \textit{robust mathematic results}
- \textit{efficient computational algorithms}
- under the interest of a broad community

The use of wavelet techniques has exponentially grown, since late 80’s

[Jaffard, Meyer, Ryan (2001), Meneveau (91), Chen (83), Morlet (83)].
The more popular characteristic of the wavelet techniques are the introduction of the time-scale decomposition.

Musical structure => events localized in time.

A piece of music can be understood as a set of musical notes characterized by four parameters:

- frequency, time of occurrence, duration and intensity

[Domingues(2005), Daubechies(92), Lau&Weng(95), Farge(92)].
Continuous wavelet transform (CWT)

CWT of a time series $f$ is defined by

$$\mathcal{W}_f(a, b) = \int_{-\infty}^{\infty} f(u) \overline{\psi}_{a,b}(u) \, du \quad a > 0,$$

where

$$\psi_{a,b}(u) = \frac{1}{\sqrt{a}} \psi \left( \frac{u - b}{a} \right)$$

represents a chosen wavelet function family, named mother-wavelet.
Summary

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Atmospheric applications

CWT

Preliminary Results

CWT

Can be used in the analysis of non-stationary signals to obtain:

- Information on the pseudo-frequency or scale variations
- The detection of structures localization in time and/or in space.
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Morlet wavelet

Analysis

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CWT - when scale and localization parameters assume continuous values.

A wavelet function must satisfy the following conditions.

1) The integral of the wavelet function, usually denoted by \( \psi \), must be zero. This assures that the wavelet function has a wave shape and it is known as the admissibility condition.

2) The wavelet function must have unitary energy. This assures that the wavelet function has compact support or has a fast amplitude decay (in a physical vocabulary \( e\text{-folding time} \)), warranting a physical domain localization.
Examples: CWT

Amplitude modulation
Frequency modulation
Abrupt changes in time
Morlet wavelet

It is formed by a plane wave modulated by a gaussian function and it is given by

$$\psi(x) = \pi^{-\frac{1}{4}} \left( e^{i\xi x} - e^{-\frac{\xi^2}{2}} \right) e^{-\frac{x^2}{2}},$$

where $\xi$ is a non dimensional value.
Morlet wavelet - real part
Methodology:

- short and long range Eta model runs
- an observation station data sets
- during part of a summer/fall season
- analysis of variance wavelet: scalogram
- using the continuous wavelet transform with Morlet mother-wavelet, family 6.
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Morlet wavelet Analysis
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Air Temperature (2 meters)
Precipitation (mm/day)
Relative Humidity (%)
Zonal wind (m/s)
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Meridional wind (m/s)
Next steps!

- To use more features of this wavelet, as the **phase** and the global wavelet aspects.
- To identify why could be the causes of these differences;
- To study if this behaviour is representative in space:
  - Using a two or three dimensional transform - time-space multiscale analysis.
Other examples: automatic mesh refinement
Other examples: turbulence analysis
Obrigada! Thanks!
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