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ON A BRIEF REVIEW OF NONLINEAR DYNAMICS OF A FREE-FREE BEAM IN SPACE AND EXCITED FOR GRAVITY POTENCIAL

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Abstract: The coupled nonlinear pitch-bending responses of a free-free beam in a circular orbit, when the beam is subjected to a periodic external excitation, are analyzed. The nonlinearities present in the governing equations of motions are both due to deformation of the beam (i.e. curvature and inertia nonlinearities) and to the gravity-gradient moments, due to central attraction massive body. Multiple Scale Perturbation method was used to analyze the governing equations of motion. Several resonant motions exhibited by the system are analyzed in details, namely, harmonic resonances when the frequency of the external excitation, Ω, is either near the natural frequency of the flexural or of the pitch motion, and a superharmonic resonance when Ω is near one half of the natural frequency for the pitch motion. The latter two resonances are associated with very low excitation frequencies. We also showed some differences when we consider or not the presence of the nonlinearities due to curvature of the beam in orbit in superhrmonic resonance.

Keywords: Orbit beam theory, Perturbation Technique

1. INTRODUCTION

The dynamic behavior of a flexible beam is of increasing interest on engineering sciences, because it is useful on some mathematical models those are useful to dynamics analysis of blades of helicopters, robotic manipulators, and antennas in spacecrafts, flexible satellites and other systems that have flexible and angular movements. For a beam in orbit, gravity gradient effects make the beam oscillate relatively to an orbital referential with pitch frequencies (movement around of the normal orbit) those are the same size of the orbital angular velocity. There are other forces those acting on objects on space, that same be small, they modified along of the time the orbital elements. An example of these forces, we may mention the solar radiation disturbance [1] and the atmospheric drag [2]. These two forces are illustrated in Fig. 1. The effects of these forces are not considered in this work. We will consider the gravity gradient torque only. The linear formulation shows that the vibrations of an isotropic beam in two principal planes are independents, so a forced motion in a plane is always stable in this plane. In the general case when the amplitude of vibration are large, several nonlinear effects, such as nonlinear curvature, axial inertia forces, damping and nonlinearities of materials [3], induce nonlinear terms in equations of the motion and in their boundary conditions. Several authors studied flexible spacecraft models [4]-[8], that were approximate for free-free beams, as well as models of spatial vehicle. In version of rigid body including articulate flexible appendages. Several of these studies include a linearization of the equations of the motion.

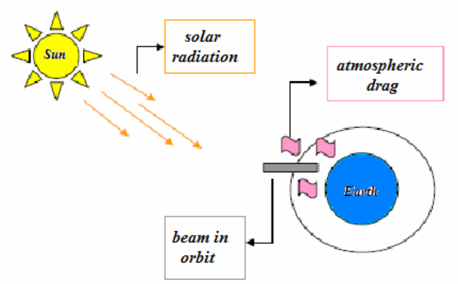

Fig. 1. Illustration of pressure radiation solar and atmospheric drag acting on beam in orbit

2. MATHEMATICAL MODEL AND GOVERNING EQUATIONS OF MOTION

The system considered here, consists of a free-free homogeneous beam of length L and constant specific mass m, and stiffness D_ζ, whose center of mass C is in a circular orbit around a center of attraction E.

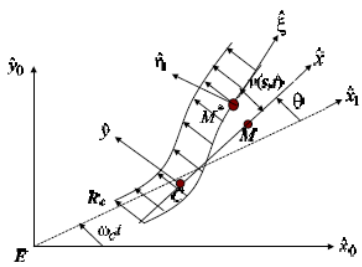

Fig. 2. Free-free beam in circular orbit

As shown in Fig. 2, the motion is described in terms of the elastic deformation $v(s, t)$ (normalized by the length of the beam) and of the pitch angle $\theta(t)$ between a "local vertical" and a principal axis of the deformed beam. The quantities s and t are, respectively, arc-length along the beam, normalized by L, and mormalized time. The variables and nomenclatures used here are the same as those used by [3]. Let the beam be subjected to a distributed periodic force $F_\eta(s,t) = E_\eta(s)\cos(\Omega t)$ that is applied along of the $\hat{\eta}$ direction shown in Fig. 2. with $v(s,t)$ approximated as $v(s,t)=F(s)v_t(t)$, and dots used to denote differentiation with respect to normalized time t, the $O(\varepsilon^3)$ normalized differential equations of motion are:

$$
\begin{aligned}
&\ddot{v}_t + cv_t + \omega^2 v_t + (\beta_1 - 1)(2\omega_c + \dot{\theta})\dot{\theta}v_t - \\
&-3\omega_c^2(\beta_1 + 1)\theta^2 v_t + \beta_2 v_t \left(v_t^2\right)^{\cdot\cdot} + \beta_3 v_t^3 = \\
&\quad = f_v \cos(\Omega t) + v_t^2 f_{v\eta} \cos(\Omega t)
\end{aligned}
\tag{1}
$$

$$
\begin{aligned}
&\ddot{\theta} + 3\omega_c^2\theta - 2\omega_c^2\theta^3 - 12\beta_1 \Big[v_t^2\left(\ddot{\theta} + 3\omega_c^2\theta\right) + \\
&+ \left(\omega_c + \dot{\theta}\right)\left(v_t^2\right)^{\cdot}\Big] + 12\left[\left(\omega_c + \dot{\theta}\right)v_t^2\right]^{\cdot} - 36\omega_c^2 v_t^2\theta = \\
&\quad = f_\theta \cos(\Omega t) + v_t^2 f_{\theta\eta} \cos(\Omega t)
\end{aligned}
\tag{2}
$$

The full governing differential equations of motion were expanded so those perturbation methods can be used to analyze de motion. In equations (1) and (2), c is a structural damping coefficient, normalized by $L^2/\sqrt{m/D_\zeta}$, ω is the undamped natural frequency of the flexural motion and ω_c is the angular velocity of the circular orbit of the beam's mass center; both were normalized by the quantity $L^2\sqrt{m/D_\zeta}$.

The quantities $\beta_1, \beta_2, \beta_3$ are Galerkin coefficients defined in [4].

The value of the constants ω, β_1, β_2 and β_3 for the first mode of oscillation, are given in Table 1 for several values of ω_c. Note that for $0 < \omega_c \le 1$ the values of the constants shown are within 1% of their values for the limiting case $\omega_c = 0$ (which corresponds to a free-free beam that is not in orbit, as found in classical structural mechanics textbooks).

Table 1. Values of ω, β_1, β_2 and β_3 for the first mode of a beam in circular orbit

ω_c	ω	β_1	β_2	β_3
0	22.373	3.0498	61.2	20 581
0.02	22.373	3.0498	61.2	20 581
1	22.577	3.0496	61.2	20 689
5	27.03	3.0463	61.208	23 306

The Table 2 shows the corresponding values of $f_v, f_{v\eta}, f_\theta$ and $f_{\theta\eta}$ for several illustrative functions $E_\eta(s)$, with $F(s)$ equal to the eigenfunction for the first bending mode and $0 \le \omega_c \le 1$.

Table 2. Values of $f_v, f_{v\eta}, f_\theta$ and $f_{\theta\eta}$ for $0 \le \omega_c \le 1$ and ω for the first bending mode

$E_\eta(s)$	f_v	$f_{v\eta}$	f_θ	$f_{\theta\eta}$
F(s)	1	44.96	0	6.07x10-7 ≈ 0
1	0	-54.34	0	≈ 0
s-1/2	0	-12.3	1	-37.9
(s-1/2)2	-0.07247	-7.87	0	≈ 0
s2	-0.07247	-33.76	1	-37.9

Equations (1) and (2) exhibit a number of resonance conditions involving the natural frequencies ω and $\omega_\theta = \sqrt{3}\omega_c$, and the frequency Ω of the external excitation. This include the internal resonances $\omega_\theta \approx \omega$ and $\omega_\theta \approx 2\omega$. However, these internal resonances are not physically possible due to the fact that $\omega > \omega_\theta$.

To analyze the coupled motions governed by equations (1) and (2), we used the multiple scales method (see [4]).

2.1. Super harmonic resonance with Ω *near* $\sqrt{3}\omega_c/2$

The amplitude-frequency relationship for the steady-state motion (i. e. A_θ = constant $\equiv A_{\theta_e}$ and $\gamma =$ constant $= \gamma_e = \pi/2$ or $-\pi/2$) is:

$$
\begin{aligned}
&6\varepsilon\sigma_1 \pm \frac{\alpha_1\left(1 - \frac{1}{2}\varepsilon\sigma_1\right)\left(\varepsilon f_{v1}\right)^2}{\varepsilon A_{\theta_e}} + \\
&+ \varepsilon^2\left[\frac{3}{2}A_{\theta_e}^2 + \frac{16}{27}\left(\frac{f_{\theta 1}}{\omega_c^2}\right)^2 + \alpha_2 f_{v1}^2\right] = 0
\end{aligned}
\tag{3}
$$

The pitch motion for this case consists of an oscillation with amplitude equal to, and frequency, superimposed on an oscillation with amplitude and frequency equal. The amplitude of the component with frequency depends on the nonlinearities and is determined by equation (3).

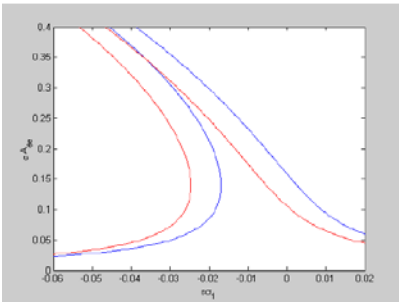

Fig.2. Amplitude-frequency pitch response for Ω near $\sqrt{3}\omega_c/2$, with $0 \le \omega_c \le 1$ $(\omega \approx 22.373)$ and $\varepsilon f_{v1}/\left(\omega^2 - 3\omega_c^2/4\right) = 0.02$ $\left(\approx 10/22.373^2\right)$

2.2. Primary resonance with Ω *near* ω

The amplitude-frequency relationship for the steady-state bending response of the beam is obtained by solving the conditions for elimination of secular terms [4]. If $f_{v\eta 1} A_{v_e}^2 / 4 << f_{v3}$, the amplitude-frequency relationship for the harmonic response when Ω is near ω is essentially given as shown in equation (4) bellow:

$$(\omega c_2)^2 + \left\{ 2\omega^2 \sigma_v + 3\omega_c^2 \left[1 + \frac{2\omega_c^2 (1-\beta_1)^2}{4\omega^2 - 3\omega_c^2} \right] A_\theta^2 + \right.$$
$$\left. + \left[\beta_2 \omega^2 - \frac{3}{4}\beta_3 - \frac{24\omega_c^2 \omega^2 (1-\beta_1)^2}{4\omega^2 - 3\omega_c^2} \right] A_{v_e}^2 \right\} \approx \left(\frac{f_{v3}}{A_{v_e}} \right)^2 \quad (4)$$

The values of the coefficients of A_θ^2 and $A_{v_e}^2$ for the first mode are:

$0 \le \omega_c \le 1$: $\quad A_\theta^2 \approx 3\omega_c^2$ and $\quad A_{v_e}^2 \approx 1.5\text{x}10^4$

$\omega_c = 5$: $\quad A_\theta^2 = 80.53$ and $\quad A_{v_e}^2 = 26\,506$

Since the value of the coefficient of $A_{v_e}^2$ is much higher than that for the coefficient of A_θ^2, the amplitude-frequency response for the directly excited bending motion is essentially the same as the classical response of a Duffing oscillator with a softening nonlinearity.

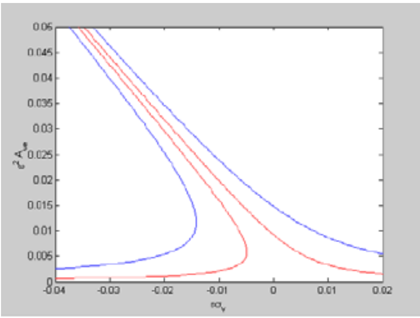

Fig.3. Amplitude-frequency bending response for Ω near ω, with $\omega_c = 1, \omega = 22.577$ (or $0 \le \omega_c \le 1$) and $\varepsilon^2 c_2 = 0.05$

2.3. Primary resonance with Ω near $\sqrt{3}\omega_c$

For this case the equilibrium solution is given by:
Equilibrium E_1:

$$\sin \gamma_{\theta_e} = 0 \quad \therefore \quad \cos \gamma_{\theta_e} = \pm 1 \quad (4)$$

$$\sigma_\theta = -\frac{1}{4} A_{\theta_e}^2 - \frac{\alpha_3 + \alpha_4}{6} f_{v1}^2 \pm \frac{f_{\theta 3} + \alpha_5 f_{\theta\eta 1} f_{v1}^2}{6\omega_c^2 A_{\theta_e}} \quad (5)$$

Equilibrium E_2:

$$\cos \gamma_{\theta_e} = -\frac{1}{A_{\theta_e}} \left[\frac{f_{\theta 3}}{2\alpha_4 f_{v1}^2} + \frac{\alpha_5}{2\alpha_4} f_{\theta\eta 1} \right] \quad (6)$$

$$\sigma_\theta = -\frac{1}{4} A_{\theta_e}^2 + \frac{\alpha_4 - \alpha_3}{6} f_{v1}^2 \quad (7)$$

The pitch amplitude-frequency response curves for equilibrium E_1 and E_2 given by equations (5) and (7) are shown in Fig. 4 for $\omega_c = 1, \omega = 22.577$ (first bending mode), f_v=15 and several values of the parameter $f = \left(f_\theta + \alpha_5 f_v^2 f_{\theta\eta} \right) / \omega_c^2$ .Equilibrium E_2 exists only in the region where the values of $|\cos \gamma_\theta|$ determined from equation (6) are not greater than unity

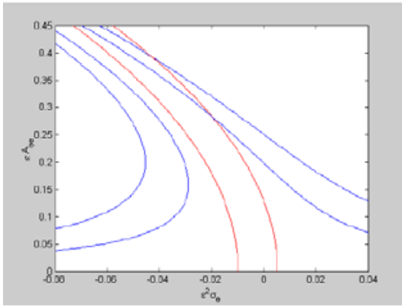

Fig. 4. Amplitude-frequency pitch response for Ω near $\sqrt{3}\omega_c$, with $\omega_c = 1$ (or $0 \le \omega_c \le 1$), $\omega_c = 1, \omega = 22.577$, $f_v = 15$ and several values of $f = \left(f_\theta + \alpha_5 f_v^2 f_{\theta\eta} \right) / \omega_c^2$

3. CONCLUSIONS

In this paper we formulated the nonlinear differential equations, mathematically consistent, governing the coupled of motion pitch-bending for a beam in orbit. The formulation used here related the nonlinear dynamic of beam carrying out account every the geometric nonlinearities in the system, beyond nonlinearities due orbital effects. The complete nonlinear equations, for a beam in orbit, was expanded for include every nonlinearities thus cubic order in an account parameter ε .

The material that constitute the beam was assumed be linear and therefore, the nonlinearities due to the deformations was caused by changing in geometry of the system. This deformations include nonlinearities of inertia, and nonlinear terms due curvature of the beam. The equations also contain second and third degree, i. e., $O(\varepsilon^2)$ e $O(\varepsilon^3)$, of the nonlinear terms of the a coupling between the pitch and bending motions of the beam. Some terms in equations of the motion are increased by de Galerkin coefficients $\beta_1, \beta_2, \beta_3$. Nonlinear equations of the motion formulated and expanded to include polynomials nonlinearities of third order were applied to study the resonance of the pitch-bending coupling to the beam in circular orbit about the center of mass of the attractor body. The nonlinearities in equations are due nonlinear curvature and inertia effects, as well as due to the pitch-bending coupling and of contribution of gravity gradient moment. We considered three types of resonances: super harmonic pitch resonance, primary bending resonance and primary pitch resonance. For the super harmonic pitch resonance it was found that the first approximation for the pitch response consists of two harmonic components, with the amplitude of one of the components being affected by the nonlinearities. For the primary bending resonance it was determined that while the amplitude-frequency response of the bending motion is characteristic of a classical Duffing oscillator, the pitch component of the response consists of a low frequency oscillation whose amplitude is dependent on initial conditions and a higher frequency component whose amplitude is dependent on the steady-station bending amplitude. For the primary pitch resonance the pitch response was shown to exhibit characteristics of a Duffing oscillator with a softening nonlinearity and a parametrically excited Duffing oscillator. It was also found that if the pitch motion is started with small initial conditions within a certain region in space, the pitch motion will grow to a maximum value which is independent of the pitch initial conditions [3].We showed that internal resonances are not physically possible, because any natural frequency is always greater than the pitch natural frequency. The equation of the motion involved the inclusion of moments and products of inertia, including terms that are originated of expression of beam's curvature. They contained too nonlinear terms terms originated of gravity gradient. Therefore, we take care to the nonlinearities were remove of the formulation of problem to de consistent way. The linearization of the equations of motion around of the equilibrium configuration, calling gravity gradient stabilization, was really a reasonable hypothesis, inasmuch as the disarranging with vertical local and elastic flexion was small. An interesting review concerning this subject was done by [9]. Finally we mention that the numerical simulations show the efficiency of the theory revised in this paper. We restrict the results to super harmonic case.

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APPENDIX. SOME NUMERICAL SIMULATIONS: SUPER HARMONIC CASE

Considering equations (1) and (2) and taken into account the time interval **[**0 90] and Initial Conditions [0 0 0 2]

$$f_v = 1,\ f_{v\eta} = 44.96,\ \ f_\theta = 0,\ \ f_{\theta\eta} = 0$$

$$\beta_1 = 3.0463,\ \beta_2 = 61.208,\ \beta_3 = 23306$$

$$\omega_c = 5,\ \ \Omega = \frac{\sqrt{3}\omega_c}{2},\ \ \omega = 27.03,\ \ c = 0.5$$

We will obtain by numerical integration the Fig 4a, b, c, d, e, f, and g

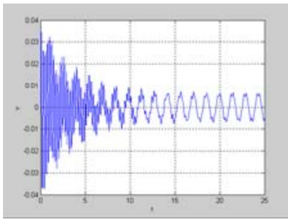

Fig.4a. Time history of v

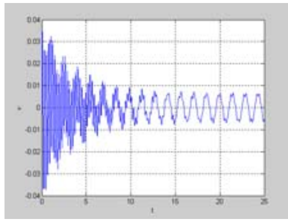

Fig.4b. Time history of $\dot{v}$

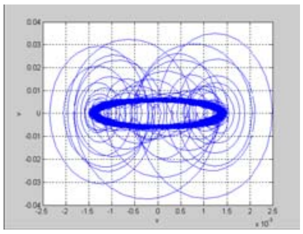

Fig.4c. Phase portrait v vs. $\dot{v}$

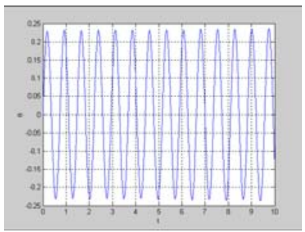

Fig.4d. Time history of θ

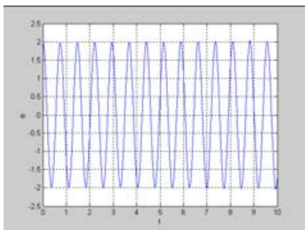

Fig.4e. Time history of $\dot{\theta}$

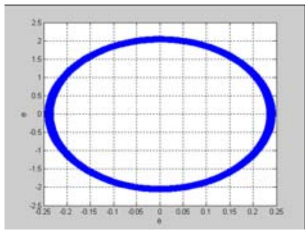

Fig.4f. Phase portrait θ vs. $\dot{\theta}$

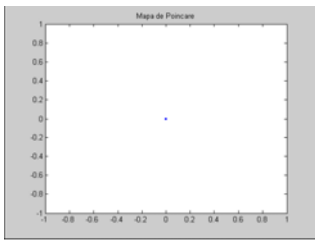

Fig.4g. Poincaré's map