Analysis of fine-scale canopy turbulence within and above an Amazon forest using Tsallis’ generalized thermostatistics

Maurício J. A. Bolzan, Fernando M. Ramos, Leonardo D. A. Sá, Camilo Rodrigues Neto, and Reinaldo R. Rosa

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We analyzed the probability density function (PDF) of velocity and temperature differences in the canopy sublayer of Amazonia based on Tsallis’ generalized thermostatistics theory. We show that such a theory provides an accurate framework for modeling the statistical behavior of the inertial subrange above and below the canopy. For this, we compared the experimental PDFs with the theoretically predicted ones. The data were measured during the wet season of the Large-Scale Biosphere-Atmosphere Experiment in Amazonia (LBA), which was carried out during the months of January–March 1999 in the southwestern part of Amazonia region. Measurements were made simultaneously at different heights in a 60 m micrometeorological tower located in the Biological Reserve of Jaru (10°04’S, 61°56’W), Brazil. The fast response wind speed measurements, sampled at 60 Hz rate, were made using three-dimensional sonic anemometers at the heights of 66 m (above the canopy) and 21 m (below the canopy). The results showed good agreement between experimental data measured above the canopy forest and Tsallis’ generalized thermostatistics theory. For below canopy data, the agreement between experimental and theoretical PDFs was fairly good, but some distortion was observed. This is probably due to some peculiar characteristics of turbulent momentum transfer process inside the forest crown. Discussion is presented to explain these results. Conclusions regarding the absence of “universal scaling” in the inertial subrange are also presented in the context of the entropic parameter of Tsallis’ theory.

INDEX TERMS: 3379 Meteorology and Atmospheric Dynamics: Turbulence; 3322 Meteorology and Atmospheric Dynamics: Land/atmosphere interactions; 3307 Meteorology and Atmospheric Dynamics: Boundary layer processes; 3210 Mathematical Geophysics: Modeling; 3374 Meteorology and Atmospheric Dynamics: Tropical meteorology; KEYWORDS: Turbulence, Amazonian forest, PDF, intermittency, Tsallis’ entropy


1. Introduction

One aspect of the vegetation–atmosphere interactions that is complex and not yet completely solved is the turbulent exchange of momentum and scalars. This is due, at least partially, to the fact that turbulence is a “high-dimensional” phenomenon [Lumley, 1992; Frisch, 1995]. As was mentioned by Speziale [1991], while most researchers agree that the basic physical aspects of the mechanically generated turbulence can be described by the Navier–Stokes equations, limitations in computer capacity make it impossible to directly solve these equations for high Reynolds numbers turbulent flows (fully developed turbulence). The turbulence modeling may be present in a variety of forms: Reynolds stress models; subgrid-scale models for large-eddy simulations (LES); spectral models; and probability density functions (PDF) models. PDFs models are useful to represent turbulent fluctuation fields [Frisch, 1995], to investigate scaling properties of the turbulence [Tcheou et al., 1999], and to evaluate subgrid representation of turbulent behavior in LES modeling schemes [Meneveau, 1994].

Traditionally, the properties of turbulent flows are studied from PDFs of velocity differences \( v_r(x) = v(x) - v(x + r) \) at different scales \( r \). As happens with other physical systems that depend on the dynamical evolution of a large number of nonlinearly coupled subsystems, the spectral energy cascade in turbulence generates a spatial scaling behavior (power law behavior with \( r \)) of the moments \( \langle v_r^n \rangle \) of the \( v_r \) PDF (the angle brackets \( \langle \rangle \) denote the mean value of the enclosed quantity). As discussed by many authors [Chu et al., 1996; Högström and Bergström, 1996], at large scales of fully developed turbulent flows, the PDFs are normally distributed, but far from this domain...
they are strongly non-Gaussian and display tails fatter than expected for a normal process [Monin and Yaglom, 1975; Kevlahan and Vassilicos, 1994; Höögström and Bergström, 1996]. This is the striking signature of the intermittency phenomenon. As was mentioned by Frisch [1995] and Sreenivasan and Antonia [1997], many PDFs models have been proposed to explain this feature. Most of these models, are used for isotropic inertial subrange turbulence fluctuations, and are based on Kolmogorov hypotheses as well as their refined versions [Kolmogorov, 1991; Kraichnan, 1991; Hill, 1997], and contain the energy cascade phenomenology, such as in the lognormal [Kolmogorov, 1962], multifractal [Parisi and Frisch, 1985; Tchéou et al., 1999], log-Poisson [She and Waymire, 1996] models. Another way to investigate PDFs functions is by means of Generalized Thermostatistics concepts [Tsallis, 1988; Ramos et al., 1999]. This approach is used in this work to study PDFs obtained above and below the Amazonian forest canopy.

Since it was first proposed by Ramos et al. [1999], the connection between Tsallis’ generalized thermostatistics and turbulence is now attracting a growing interest [Bolzan et al., 2000; Beck, 2000; Arimitsu and Arimitsu, 2000; Ramos et al., 2001]. Nevertheless, many aspects related to this new approach are still not well understood, particularly concerning the influence of coherent structures and intermittency on the statistical distribution of turbulence fluctuations. The objective of this study is to investigate the validity of this general PDF approach to velocity and temperature differences in the canopy sublayer of Amazonia. For this purpose, we use fast response turbulent data, measured above and below an Amazonian forest canopy.

The paper is organized as follows. Section 2 is devoted to the theoretical background of this new model. Section 3 is concerned with the measurements and experimental site. Section 4 presents and discusses the results. Finally, in section 5, we present the conclusions.

2. Theory

Based on the scaling properties of multifractals, a generalization of Boltzmann–Gibbs thermostatistics has been proposed [Tsallis, 1988; Tsallis et al., 1995] through the introduction of a family of nonextensive entropy functionals, $S_q(p)$ with a single parameter $q$. These functionals, derived for Levy processes, reduce to the classical extensive Boltzmann–Gibbs form as $q \to 1$. Optimizing $S_q(p)$ subject to appropriate constraints [Tsallis et al., 1995], we obtain the distribution:

$$p_q(x) = [1 - \beta (1 - q)x^q]^{1/(1 - q)}/Z_q$$

where $Z_q$ is a normalization factor. In the limit of $q \to 1$, we recover the Gaussian distribution. Ramos et al. [1999] have shown that, assuming $x \equiv v_r$ in (1), provides a simple and accurate model for handling the PDF problem. Equation (1) is similar to that of a Levy Process. Hence, one can state that Levy flights through Tsallis’ theory for turbulent diffusion is the phenomenological analog of Brownian motion for molecular diffusion [Tsallis et al., 1995].

In this paper, we will adopt a generalization of the PDF model used in our previous works [Ramos et al., 1999; Bolzan et al., 2000], assuming that $p_q(v_r)$ is given by [Beck et al., 2001]:

$$p_q(v_r) = \left[1 - \beta (1 - q)\left(|v_r|^{2q} - C \text{sign}(v_r)ight) \cdot \left(|v_r|^{2q} - \frac{1}{3}|v_r|^{2q}\right)^{1/(1-q)} / Z_q\right]$$

where $C$ is a small skewness correction term, and $Z_q$ is given by

$$Z_q = \langle |v_r|^n \rangle = \frac{a_n^{q+1}}{\gamma} B(\phi_0, \chi_0)$$

with $B(\phi_0, \chi_0) = \Gamma(\chi_0)\Gamma(\phi_0 + \chi_0)$, $\phi_0 = (1 + m_0)/2$, $\chi_0 = \ell - \phi_0$, $l = 1/(q - 1)$, $m_0 = (1 - \alpha)/\alpha$, and $a = s/\alpha$.

Neglecting the skewness correction term, we obtain for the PDF $n$th moment:

$$\langle |v_r|^n \rangle = \frac{a_n^{q-n} B(\phi_0, \chi_0)}{B(\phi_0, \chi_0)}$$

The parameters $q$ and $\beta$ determine the shape of the PDF and can be computed from (4), where the values of the variance, $\langle |v_r|^2 \rangle$, and the kurtosis, $K_r = \langle |v_r|^4 \rangle / \langle |v_r|^2 \rangle^2$ are measured at each scale. The parameter $q$ depends on $K_r$ through the equation:

$$K_r = \frac{B(\phi_2, \chi_2)B(\phi_0, \chi_0)}{B(\phi_2, \chi_2)^2}$$

Particularly for $\langle |v_r|^2 \rangle = 1$, $\beta$ is given by

$$\beta = \frac{\ell B(\phi_2, \chi_2)}{B(\phi_0, \chi_0)}^{1/\alpha}$$

we remark that the entropic parameter depends only on $K_r$ (while, for instance, the variance is a function of $q$ and $\beta$). It is well known that large values of $K_r$ corresponds to the presence of strong bursts in the kinetic energy dissipation rate and is a signature of intermittency [Sreenivasan and Antonia, 1997]. Thus, $q$ can be used as a measure of intermittency in turbulent flows. We also note that if we assume a scaling of the moments $\langle |v_r|^n \rangle$ of $v_r$ as $r^{\gamma n}$ (which is valid for inertial subrange scales, for sufficiently high Reynolds number) the scale variation of $q$ and $\beta$ can be determined from (4) and (5), extrapolating the experimental values of $\langle |v_r|^2 \rangle$ and $K_r$ at a given reference scale (say, the Kolmogorov scale, $\eta$). Moreover, this extrapolation procedure can be extended over a much wider range of scales with respect to the inertial subrange using the concept of extended self-similarity proposed by Benzi et al. [1995]. This approach requires to numerically solve the Kolmogorov equation using, as initial condition, the observed value of $\langle |v_r|^2 \rangle$ at the reference scale.

3. Data and Experimental Site

The above mentioned turbulence statistical model was tested with data obtained during an intensive micrometeorological campaign, a part of the wet season Large-Scale Biosphere-Atmosphere Experiment in Amazonia (LBA) project. The experiment was carried out during the months of January–March 1999. Measurements were made
simultaneously at different heights in a micrometeorological tower located in the Biological Reserve of Jaru (Rebio Jaru: 10°04’S, 61°56’W), Brazil. These heights are: Upper Level (66 m—above the canopy); Medium Level (45 m—at the canopy top); and Lower Level (21 m—below the canopy). The fast response wind speed measurements, sampled at 60 Hz, were made using three-dimensional sonic anemometers (Campbell Scientific Inc., model CSAT-3-L60) and sonic thermometers (Campbell Scientific Inc., model CA27).

[11] During the experiment, the state of the atmosphere was characterized by the existence of strong convective activity in the diurnal period with isolated rain events and also, by the influence of the Convergence Zone of Southern Atlantic which was active above the Brazilian State of Rondônia (southwestern Amazonia) [Silva Dias et al., 2002]. The experimental site was characterized by the existence of reasonable fetch conditions. The area where the tower was built is surrounded by the Amazon Forest in a radius of at least 800 m around the tower. The topography of the area is not totally homogeneous, since at the southern and eastern sides of the tower there are small hills with heights of some dozens of meters. More information on the variability pattern of the main micrometeorological variables during the period of the experiment is presented by Silva Dias et al. [2002]. Culf et al. [1996] presented a discussion of the micrometeorological characteristic of the Rebio Jaru site area during a dry season.

[12] We used longitudinal (u) and vertical (w) wind velocity components as well as temperature (T) time series measured at two heights, above and below canopy (higher and lower levels, respectively), at two time intervals, 1200–1300 and 2300–0000 h, under unstable and stable conditions, respectively. Further, we applied Vickers and Mahrt’s [1997] quality control procedure for all data used in this paper. In particular, we checked the vertical velocity power spectrum, which displayed a sizable scaling range with a slope of approximately −5/3 in the inertial subrange, as shown in Figure 1. The subrange scale ends are 0.3 Hz and 7 Hz, approximately. These values have been calculated by means of Kulkarni et al.’s [1999] method based in determining isotropy coefficient using the Haar wavelet transform.

[13] We investigated the validity of Taylor’s hypothesis by means of Wyngaard and Clifford’s [1977] turbulence intensity test. Our results show that this condition holds for all analyzed data and the maximum turbulence intensity $I_{u/w}$ ($I_{u/w} = \sigma_{u/w}(\bar{U})$), where $\sigma_{u/w}$ is the standard deviation of $w$ or $u$, respectively, and $\bar{U}$ is the wind mean velocity) calculated was 0.14, lower than 0.3, the threshold value for validity of this hypothesis. However, after Hsieh and Katul [1997], this test was not necessary for structure functions in the inertial subrange (as in our paper) because they are not sensitive to the degree of deviation from Taylor’s hypothesis in this scale range, and in practice Wyngaard and Clifford’s [1977] corrections are not necessary at all in the inertial subrange.

4. Results and Discussions

[14] In order to validate the model described in section 2, we compared measured $w_r$ distributions, corresponding to two different heights (above and below the canopy, respectively), with the theoretical PDFs obtained from (2). For each data set, we measured the variance and the kurtosis, which allowed us to compute $q$ and $\beta$ by means of the corresponding expressions obtained from (4). The parameter $\alpha$ was chosen according to the empirical formula $\alpha = 6 - 5q$; a somewhat similar expression ($\alpha = 2 - q$) yielded excellent results in fitting histograms from a Couette–Taylor flow experiment [Beck et al., 2001]. In spite of the fact that this is a controversial subject [Kaimal et al., 1972; Sreenivasan and Antonia, 1997], we accepted the assumption of frozen turbulence which is expressed by the well-known Taylor’s hypothesis [Taylor, 1935] to obtain length distance intervals from time intervals: $r = \bar{U} \Delta t$, (where $\bar{U}$ is the mean velocity of the flow which impacts the measurement instruments). This procedure has been successfully used by authors as Katul et al. [1994a] to analyze inertial subrange turbulent fluctuations of micrometeorological variables in the atmospheric surface layer and, as was mentioned in our earlier section, Taylor’s hypothesis holds for our data.

[15] Figures 2 and 3 present the theoretical and experimental semilogarithmic plots of $p_q(v_r)$ versus $v_r$ at four different scales, properly rescaled and vertically shifted for better visualization. Figure 2 data was measured above the canopy, probably inside the transition sublayer. Figure 3 data was measured approximately 15 m below the canopy top. Overall, we observe that the theoretical results (solid lines) are in good agreement with measurements across spatial scales spanning three orders of magnitude and a range of up to 10 standard deviations, including the rare fluctuations at the tail of the distributions. The transition from large-scale Gaussian behavior to a power law form as $r$ decreases is quite evident and well reproduced by Tsallis’ distribution. At small scales, the distributions have tails larger than that expected for a normal process. This excess of large fluctuations, compared to a Gaussian distribution, is a well-known signature of intermittency. The spiky shape near the origin is also indicative of intermittency [Sreenivasan and Antonia, 1997]. We obtained similar agreements for the PDFs of $u$ and $T$ (not shown in the text).
It is important to stress that the proposed PDF model was validated with data measured in a "noisy" real atmospheric surface layer, close to a very complex surface, the Amazon forest. Thus, it is worthwhile to ask how general is the model described by (2). To answer this question, we have to admit that a generic intermittent field presents a universal behavior and that the peculiar flow immediately above Amazon forest canopy presents a universal behavior, too. It is difficult to experimentally determine whether such a proposition is true or not. As stressed by Frisch [1995], the exact probabilistic formulation of a PDF requires reliable measurements at the smallest scales of the flow and this is still very difficult to obtain with the available wind velocity measuring probes. In fact, there is growing evidence that there are structures with nontrivial geometry down to very fine scales, maybe of the order of the Kolmogorov microscale [Frisch, 1995].

In the present context, many factors contribute to make the above question even more difficult to answer. First, the structure of the turbulent flow close to forest canopies presents an inflectional point in the mean longitudinal velocity profile. In such situations, the flow displays some very peculiar features [Raupach et al., 1996], caused by the so-called inflectional instabilities. These instabilities generate "roll-shaped" structures whose dissipation is slower comparatively to "normal" dissipative structures [Robinson, 1991]. Second, the wet season disturbed tropical boundary layer shares some similarities with tropical marine boundary layer [Garstang and Fitzjarrald, 1999] which presents characteristic "updrafts" and "downdrafts" structures [Wyngaard and Moeng, 1992], many times associated with the action of individual eddies [Sun et al., 1996]. The dissipation processes that occur at the edge of individual eddies present very complex and peculiar features, as stressed by Shaw and Businger [1985] and Katul et al. [1997]. In such situations, it would be difficult to admit a general pattern for all intermittent fluctuations in the actual atmosphere. Nevertheless, as most of these intermittent events are very rare, their signatures would appear only as noise in the tails of wind-velocity histograms.

Comparing the histograms of Figures 2 and 3, we note that the kurtosis is consistently higher below the canopy under diurnal conditions, regardless the observed scale. This and other features are highlighted in Figures 4 and 5, which depict the scale variation of parameter q, for vertical velocity (w), longitudinal velocity (u) and temperature (T), above and below canopy data, under diurnal and nocturnal conditions. Three general trends emerge from these results.

First, we observe that the entropic parameters for u, w, and T (q_u, q_w, and q_T, respectively) decrease as r grows, showing a pattern already observed [Ramos et al., 2001; Beck et al., 2001]. Katul et al. [1994b], using a parameter related to a scale kurtosis, the wavelet flatness factor (FF), have shown that in the inertial subrange scales, the more the
The momentum exchange processes between the environment and the canopy affect the turbulent transfer processes, associated with the action of the so-called coherent structures. Two main steps characterize these processes: a sweep phase and an ejection phase [Gao et al., 1989].

2. Large-eddy simulations, carried out to better understand the complex momentum transport along the canopy height and the role played by the pressure forces in each level of the crown [Shen and Leclerc, 1997], have shown that above and below the treetop, the contribution to momentum and to a lesser extent to heat transfer is mainly controlled by the sweep/ejection processes mentioned above.

3. The structure of turbulence in a forest is expected to be clearly different of the turbulence structure over smooth surfaces. In the canopy structure, there is a vertical variation of leaf density and the distribution of sources and sinks of momentum and scalars which are likely to introduce major differences from crop to crop. Also, the nature of thermal stability regime influences the exchange processes there [Shaw et al., 1988; Kaimal and Finnigan, 1994].

It is worthwhile to mention that this bias toward higher levels of intermittency found in low-level data disappears under nocturnal conditions, as shown in Figure 4. This is due to the cyclic variation in thermal stability regimes above and below the canopy along a typical day. Schematically, from a radiative transfer budget point of view, during the day dense forest canopies storage heat in their highest parts due to the short-wave incoming solar radiation flux. On the other hand, at night the forest crown looses heat by means of long-wave infrared radiation imbalance. So, during the day the above canopy region is unstable and the below canopy region is stable. At night, the stability profile is reversed and stable conditions predominate above the canopy and the regime becomes less stable and even unstable below the canopy. This cyclic process determines variations on the thermodynamic structure of the canopy, which influences the turbulent transfer processes in this environment [Shuttleworth et al., 1985; Fitzjarrald et al., 1990; Leclerc et al., 1991]. Consequently, some nocturnal very stable conditions may exist in which the mechanical production of TKE is not sufficient to undergo the buoyant destruction of TKE above the canopy and turbulence wind fluctuations may be suppressed there [Fitzjarrald and Moore, 1990]. However, below the canopy, some unstable conditions may exist and generate some thermally induced local circulations [Bosveld et al., 1999].

3. Third, we note that under both diurnal and nocturnal conditions, $q_T$ has higher values above the canopy. This could be attributed to the existence of large-scale like-ramp coherent structures in temperature fields, above and below forest canopies [Gao et al., 1989]. As has been observed by many authors, such structures are shear driven [Raupach et al., 1996], probably present an universal character after appropriate scaling [Chen et al., 1997]. These structures are also associated with the thermal gradient [Paw U et al., 1992], and are responsible for most of the transport of sensible heat downward the top canopy [Chen et al., 1997]. As the strongest shear and temperature gradients are located above the canopy, we expect that ramp-like structures will be more apparent above vegetated canopies than within them [Paw U et al., 1992]. Katul et al. [1995, 1997] have suggested that the small-scale thermal motion in the atmospheric surface layer is affected by such ramp-like organized

Figure 5. Scale variation of the parameter $q$ for longitudinal velocity ($u$) and temperature ($T$) measured above and below canopy: (a) diurnal conditions and (b) nocturnal conditions.

In fact, from a statistical point of view, below canopy turbulent turbulence presents some very peculiar features [Gao et al., 1992; Paw U et al., 1992; Kaimal and Finnigan, 1994]. Krujt et al. [2000] presented a review of some of these peculiar characteristics in their study about turbulence structure below Amazon forest, even for data of Rebio Jaru Reserve. Among the features that may impact the kurtosis, we could mention:

1. The momentum exchange processes between the atmospheric flow above and the below canopy are not continuous in time, but are characterized by intermittent

Figure 5. Scale variation of the parameter $q$ for longitudinal velocity ($u$) and temperature ($T$) measured above and below canopy: (a) diurnal conditions and (b) nocturnal conditions.
structures which may contribute to inertial subrange anisotropy in the temperature measurements. If $q_T$ is actually influenced by “large structures”, it means that the Kolmogorov cascade is “short-circuited” [Warhaft, 2000]. This means that there is a direct interaction between such structures and inertial subrange properties measured by $q_T$ as is the case with many laboratory flows [Warhaft, 2000]. Clearly, the large-scale structures are influenced by boundary conditions and through their interaction with the inertial range, influence measures of intermittency like $q$. Hence, this work clearly demonstrates that a “universal model” of turbulence intermittency may not exist for canopy flows. These findings have implications to the development of subgrid model of LES, now widely used to assess CO$_2$ exchange [Albertson et al., 2001], which are primarily based on Kolmogorov type cascades or simplistic energy backscatter corrections [Menueveau and Katz, 2000], as they are not capable to capture the effect of large-scale motion on canopy sublayer inertial range. These considerations could explain our $q_T$ results above and below canopy.

[23] Finally, we investigated via the so-called extended self-similarity approach [Benzi et al., 1995], if the results in Figures 4 and 5 are sensitive to the onset of a “real inertial subrange”. For this we numerically solved the Kolmogorov equation for $D^3(t) = (\langle |v| \rangle)^3$, and then used the appropriate scaling relations to compute $D^2(t)$ and $D^4(t)$ (for details, see Benzi et al. [1995]). This allowed us to calculate the kurtosis and through (5), the value of the entropic parameter. This theoretical values of $q$ are displayed in Figure 6 in comparison with the experimental ones (clearly, those estimated at each scale from the corresponding experimental histogram), for $u$ data measured above canopy under diurnal conditions. We observe an excellent agreement between theory and experiment that extends over a much wider range of scales with respect to the inertial subrange previously determined following the procedure described in section 3.

5. Conclusions

[24] We have studied the PDF of velocity differences in the canopy sublayer of Amazonia based on Tsallis’ generalized thermostatistics theory. We have shown that it provides a simple and accurate framework for modeling the statistical behavior of fully developed turbulence above and below the canopy forest. We also analyzed the behavior of the entropic parameter $q$ from Tsallis’ generalized thermostatistics theory. This parameter represents a flow independent measurable quantity which is robust to variations in the Reynolds number, and which can be used to objectively quantify intermittency buildup in turbulent flows. From a practical point of view, the use of the entropic parameter as a measure of intermittency is justified by the fact that $q$ is the key parameter that controls the shape of the PDF, which accurately models the statistics of turbulent velocity and temperature fluctuations. As expected by theoretical earlier results, we found the higher values of $q$ below the canopy. This is due to the peculiar characteristics of the downward kinetic energy transfer process in forest environments. Among the physical mechanisms that would be responsible by this behavior we could mention the influence of coherent structures such as in sweep/ejection motion, bluff-body forces in the crown and the peculiar thermodynamical structure of the canopy and stability conditions. A consequence of these findings is that the inertial scale scaling cannot be universal, and is influenced by large-scale flow properties (e.g., stability conditions and morphology). This has implications to the development of subgrid model of LES, which are often based on Kolmogorov cascade concepts.

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References
Chu, C. R., M. B. Parlarge, G. G. Katul, and J. D. Albertson, Probability


Kolmogorov, A. N., A refinement of previous hypotheses concerning the probability density function of the velocity increments in the fully developed turbulent boundary layer, J. Fluid Mech., 13, 82 – 86, 1962.


