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A data assimilation method used with an ocean circulation model and its application to the Tropical Atlantic

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1. Introduction

A data assimilation method based on the well known Kalman theory is considered in conjunction with the Modular Ocean Model (MOM) from NOAA GFDL. The method is applied in [illegible] along with the [illegible] surface and subsurface temperatures from the PIRATA data set.

In this paper another method for the [illegible] of the error covariance matrix is presented. It continues the studies published in the paper by authors (Belyaev K., S. Meyers and J. O'Brien 1999). The present method is applied to the GFDL primitive equation ocean model MOM 2 (Bryan 1969). The goal is to incorporate the [illegible] of the assimilation technique. This model is used only as a tool for the calculation of the thermodynamic fields, their prediction and variability. It has wide applications. Many papers discuss the model and its development. Also, the Pilot Research Moored Array in the Tropical Atlantic (PIRATA) data set is used here. This includes sea surface and subsurface temperatures records.

2. Data Assimilation Technique.

The following problem is considered. Let $\varsigma(t,\bar{x})$ be an unknown *real* which is sought in some ocean domain Ω. The model equation for ς is supposed to be written in Cartesian coordinate system as below

$$\frac{\partial \varsigma}{\partial t} = A(t)\varsigma \,,$$

where $A(t)$ is the known model operator, in general non-linear.

[illegible] define the *model* [illegible] $(t,\bar{x})$ [illegible] $t \in G$ [illegible] T [illegible] $\bar{x} \in \Omega$ [illegible] [illegible] $t \le T$ [illegible] is [illegible] to construct the optimal [illegible] [illegible] [illegible]

Let $\theta = \theta(t,\bar{x}) = \varsigma(t,\bar{x}) - \varsigma_m(t,\bar{x})$ be an error of modeling or simply error. The [illegible] supposed to be satisfied for [illegible]

$$\frac{\partial\theta}{\partial t} = A(t)\theta + \eta$$

And [illegible] the standard conditions for the noise $E\eta = 0$, $E\eta(\bar{x},\tau)\eta(\bar{y},t) = R(t)\delta(t-\tau)$ are assumed to hold. All notations are common. The function R is known.

The following problem is considered for [illegible] the optimal estimation $\hat{\varsigma}(t,\bar{x})$ [illegible] filter [illegible] unknown *real* field $\varsigma(t,\bar{x})$ satisfying the conditions:

a) $E(\hat{\varsigma} - \varsigma) = 0$

b) $E(\hat{\varsigma} - \varsigma)^2 = \min E(\tilde{\varsigma} - \varsigma)^2$ [illegible] estimation [illegible]

The optimal filter is sought as following

$$\hat{\varsigma}(t,\bar{x}) = \varsigma_m(t,\bar{x}) + \int_0^t \sum_{i=0}^{N(\tau)} \alpha(\tau,\bar{x},\bar{x}_i)\theta(\tau,\bar{x}_i)\,d\tau$$

Let $N(\tau)$ be the number of observations at time τ. In formula (6), the unknown weight-coefficients $\alpha_i = \alpha(\tau,\bar{x},\bar{x}_i)$ should be determined using condition b).

The following equation for α is

$$K(t,\bar{x},\bar{x}_j) = \int_0^t \sum_{i=0}^{N(\tau)} \alpha(\tau,\bar{x},\bar{x}_i)K(t-\tau,\bar{x}_i,\bar{x}_j)\,d\tau$$

where $K = K(t,\bar{x},\bar{y})$ is the error covariance function. Another way is suggested to define [illegible] covariance function and its time evolution. The probability distribution is found from the Fokker-Planck equation

$$\frac{\partial p}{\partial t} = -a\frac{\partial p}{\partial x} + \frac{1}{2}(R\frac{\partial^2}{\partial x^2}p)$$

where parameters are defined from model [illegible] observations. Then the covariance is calculated directly having known the probability distribution and its time evolution.

3. The numerical experiments and results

The [illegible] method has been [illegible] [illegible] analysis of PIRATA data. Some [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] model [illegible] [illegible] [illegible] [illegible] [illegible] starting from the known initial temperature salinity field described above. The real data are assimilated on the same day. Model predicted fields along with observations create the new ocean state which is taken as an initial condition for the next time step (next day). This continues during one month. For the next month the model starts again from the climatological initial state.

Let $\sigma_m^2(t)$ be the model error variance without any assimilation, i.e. the error between the model values themselves and observations. Along with the variable $\sigma_m^2(t)$ two other variances are considered: $\sigma_b^2(t)$, and $\sigma_a^2(t)$. They are the variances before and after correction respectively. The first variable shows the difference between the model values and observations at moment t, if the correction was done before at the previous time step, and the second shows the difference after the correction at the same time step.

Fig.1 illustrates the time behaviour of these variances [illegible] through March 1999 at the 10-meter depth. The top curve shows the model error variance $\sigma_m^2(t)$. It remains almost steady during the month (about 1.5 to 2 square degrees). A "spike" is observed in the end of the month due to the appearance of a warm anomaly near the equator. The model could certainly not predict or react to it. Therefore, this spike was inevitable. The second curve demonstrates the time- behaviour of variance $\sigma_b^2(t)$. One can see that in the first four days this value is even greater than the first error. It means that the model needs an "adjustment time" to come to an agreement with observations. But after four days this curve "plunges" under the first one and its values remain significantly smaller during all other days. Also, in the end of the month a similar spike is observed, but the method reacts quickly and precisely. It forces the model to hold the same level of error (around 0.5 square degrees). And at last, the lowest curve is $\sigma_a^2(t)$. Its values are almost constant (around 0.25 square degrees) throughout the period. This confirms that the method works correctly and [illegible] the observations.

This method has been compared with a [illegible] version of the Kalman-filter.

Fig. 2 shows the comparison between the two methods. The results were taken at the 40-meter level (February 99). As in Fig.1 the top curve [illegible] from [illegible] the observations. The model error [illegible] during the month [illegible] from the [illegible]

dashed-line is the error variance before assimilation of the Kalman filter version and the last curve with "circles" is the error variance of the studied method. One can see that the presented method is better. It gives not only the smallest variance, but also a quicker adjustment after unexpected spikes (in the beginning of the month and around day 20). These jumps are inevitable because the model does not have a forecast block. It is forced only by climatological winds and heat-fluxes, and it is unable to predict the real variability in the ocean. Again it should be pointed out that both assimilation methods work correctly. They both give the model adjustment to observations.

References

Belyaev K., S.Meyers, and J.J.O'Brien, 1999. Fokker-Planck equation application to data assimilation based on Kalman-filter theory. *J. of Math. Sciences*, (in press)

Bryan K., 1969. A numerical method for the study of the circulation of the World Ocean. *J. Comp. Physics*, 4, 347-376

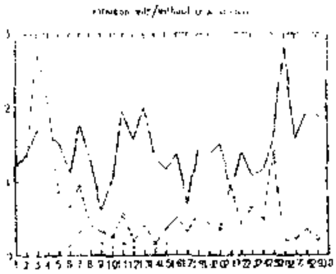

Fig.1 Variances with and without assimilation

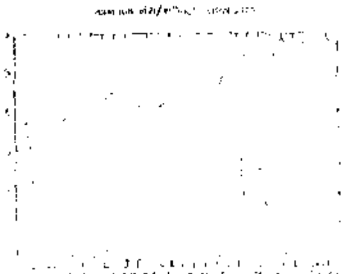

Fig. 2 Comparison of the variances with standard Kalman-filter