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RESONANTLY INTERACTING EQUATORIAL WAVES: A POSSIBLE EXPLANATION FOR THE INTRASEASONAL VARIABILITY OF THE ATMOSPHERIC CIRCULATION?

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1. INTRODUCTION

The interaction between tropical and mid-latitude atmospheric circulations and the low-frequency variability of the atmospheric flow have become topics of great importance due to the extension of the forecast period in recent years of the numerical weather prediction for weather forecasting and long-term seasonal forecasts. An important issue for understanding the global teleconnection patterns involving tropical and extratropical regions in the low-frequency range of the spectrum is how equatorially trapped modes of variability, e.g. the Madden-Julian oscillation, interact with extratropical patterns such as the Pacific South American teleconnection pattern (PSA) and the Antarctic Oscillation (AAO). This paper explores a possible explanation for the tropical-midlatitude connections on the intra-seasonal time-scales based on the resonant non-linear interaction involving equatorially trapped baroclinic Kelvin and mixed Rossby-gravity waves and barotropic Rossby waves having a significant midlatitude projection. The asymptotic method of multiple time-scales is applied to primitive equations on the equatorial β-plane in order to get a reduced model governing the weakly nonlinear interaction of the waves in a resonant triad.

2. METHODOLOGY

After some algebra, the non-dimensional primitive equations in isobaric coordinates with the equatorial β-plane approximation can be written as follows:

$$\Im\xi = \varepsilon N \qquad (1)$$

where $\xi = [u(x,y,p,t), v(x,y,p,t), \phi(x,y,p,t)]^T$, $\Im$ is the linear operator and N is the vector containing the non-linear terms, that is,

$$\Im = \begin{bmatrix} \frac{\partial}{\partial t} & -y & \frac{\partial}{\partial x} \\ y & \frac{\partial}{\partial t} & \frac{\partial}{\partial y} \\ -\frac{\partial}{\partial x} & -\frac{\partial}{\partial y} & \frac{\partial}{\partial t}\frac{\partial}{\partial p}\left(\frac{1}{\tilde{\tilde{\sigma}}}\frac{\partial}{\partial p}\right) \end{bmatrix} \qquad (2a)$$

$$N = \begin{bmatrix} -\left(u\frac{\partial u}{\partial x} + \mathrm{v}\frac{\partial u}{\partial y} + F\omega\frac{\partial u}{\partial p}\right) \\ -\left(u\frac{\partial \mathrm{v}}{\partial x} + \mathrm{v}\frac{\partial \mathrm{v}}{\partial y} + F\omega\frac{\partial \mathrm{v}}{\partial p}\right) \\ -\frac{\partial}{\partial p}\left[\frac{u}{\tilde{\tilde{\sigma}}}\frac{\partial}{\partial x}\frac{\partial\phi}{\partial p} + \frac{\mathrm{v}}{\tilde{\tilde{\sigma}}}\frac{\partial}{\partial y}\frac{\partial\phi}{\partial p} + \frac{F\omega}{\tilde{\tilde{\sigma}}}\frac{\partial^2\phi}{\partial p^2} + \frac{F\omega}{\tilde{\tilde{\sigma}}p}(1-\kappa)\frac{\partial\phi}{\partial p}\right] \end{bmatrix} \qquad (2b)$$

where ε = Ro = U / βL^2 is the equivalent for the equatorial region of the Rossby number in mid-latitudes, F = Θ/Ro, where $\Theta = \Omega / \beta L p_0$; $\kappa = R / C_p$, being R and C_p the gas constant of the dry air and the thermal capacity of the dry air at constant pressure, respectively, and $\tilde{\tilde{\sigma}} = \bar{\sigma}\, p_0^2 / \beta L^4$, where $\bar{\sigma}$ is the static stability parameter of the background state, which is characterized by a motionless, horizontally homogeneous, hydrostatic and stably stratified atmosphere. Equations (1)-(2) are nondimensionalized by using the following scaling rules:

$$(u',v') \sim O(U)\,(u,v),\ (x',y') \sim O(L)\,(x,y),\ t' \sim O(1/\beta L)t,\ p' = O(p_0)\,p,\ \omega' \sim O(\Omega)\omega,\ \phi' = O(\beta L^2 U)\phi \qquad (3)$$

Zonal periodicity and bounded solution as |y| goes to infinity constitute the horizontal boundary conditions of our problem. As vertical boundary conditions we have assumed rigid boundary conditions with no vertical flow at the surface and at a hypothetical top of the troposphere. In this way, the analysis of system (1) is done using the asymptotic perturbation method based on multiple time-scales, which is developed in terms of the small parameter ε. Thus, in the limit $0<\varepsilon<<1$ and F = O(1), both the time derivatives and the dependent variables in (1) are assumed to have uniformly valid asymptotic expansions of the form

$$\frac{\partial}{\partial t} \rightarrow \frac{\partial}{\partial t} + \varepsilon\frac{\partial}{\partial \tau} + O(\varepsilon^2) \qquad (4a)$$

$$\begin{aligned} u &= u^{(0)}(x,y,p,t,\varepsilon t) + \varepsilon u^{(1)}(x,y,p,t,\varepsilon t) + O(\varepsilon^2) \\ v &= v^{(0)}(x,y,p,t,\varepsilon t) + \varepsilon v^{(1)}(x,y,p,t,\varepsilon t) + O(\varepsilon^2) \\ \phi &= \phi^{(0)}(x,y,p,t,\varepsilon t) + \varepsilon\phi^{(1)}(x,y,p,t,\varepsilon t) + O(\varepsilon^2) \\ \omega &= \omega^{(0)}(x,y,p,t,\varepsilon t) + \varepsilon\omega^{(1)}(x,y,p,t,\varepsilon t) + O(\varepsilon^2) \end{aligned} \qquad (4b)$$

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where $\tau = \varepsilon t$ represents the long time-scale. Inserting our ansatz given by (4) into the governing equations (1) as well as into the vertical boundary conditions, it follows that the leading-order solution is written according to

$$\xi^{(0)} = \sum_a A_a(\tau)\xi_a(y)e^{ik_a x + i\varpi_a t}G_a(p) \qquad (5)$$

where the subscript **a** = (m,k,n,r) refers to a particular expansion mode characterized by a vertical mode m, a zonal wavenumber k, a meridional mode n distinguishing the meridional structure of the eigenfunctions $\xi_a(y)$ and a type of wave represented by r; r=1 for Rossby, r=2 for westward propagating inertio-gravity (WGW) and r=3 for eastward propagating inertio-gravity (EGW) waves. The mixed Rossby-gravity waves (MRGW) are associated with the n = 0 mode and are inserted in the r=1 (for $k > 2^{-1/2}$) and r = 2 (for $k < 2^{-1/2}$) categories. The Kelvin waves are represented by n=-1 and r=3. In (5), $G_a(p)$ represents the vertical eigenfunctions which distinguish the vertical structure of the linear eigenmodes. The eigenfunction associated with m = 0 has a barotropic structure, while those associated with m > 0 are characterized by a baroclinic structure. The m = 1 vertical eigenmode is characterized by only one phase inversion throughout the troposphere, occurring at the mid-troposphere, and dominates the energetics of tropical motions (DeMaria 1985; Silva Dias and Bonatti 1985). ϖ_a represents the eigenfrequencies associated with each mode **a**. The solvability condition of the $O(\varepsilon)$ problem requires that the linear inhomogeneous terms be orthogonal to the kernel of the adjoint operator of $\Im$. This condition yields the following asymptotic reduced equations governing the slowly varying wave amplitudes in a particular triad of interacting modes **a**, **b** and **c**:

$$c_a^{\,2}\frac{dA_a}{d\tau} = A_b A_c \eta_a^{bc} \qquad (6a)$$

$$c_b^{\,2}\frac{dA_b}{d\tau} = A_a A_c^* \eta_b^{ac} \qquad (6b)$$

$$c_c^{\,2}\frac{dA_c}{d\tau} = A_a A_b^* \eta_c^{ab} \qquad (6c)$$

where c_j, j = a, b or c, are the separation constants associated with each triad component and η_a^{bc}, η_b^{ac} and η_c^{ab} are the coupling coefficients. Equations (6) are valid provided that modes **a**, **b** and **c** satisfy the following resonant conditions:

$$k_a = k_b + k_c \qquad (7a)$$
$$n_a + n_b + n_c = \text{odd} \qquad (7b)$$
$$\varpi_a = \varpi_b + \varpi_c \qquad (7c)$$

3. RESULTS

A particular triad of waves satisfying (7) deserved special attention in this paper because of the significant interaction among the modes composing this triad and the possible relation of these waves to observed features in the atmospheric circulation. This triad is composed of a zonal wavenumber-2 mixed Rossby-gravity wave (mode a), a zonal wavenumber-1 Kelvin wave (mode b), both with the first baroclinic mode vertical structure, and a barotropic zonal wavenumber-3 Rossby wave having the second gravest meridional mode (mode c). Fig. 1 illustrates the energy exchange among the modes of this triad for the initial wave amplitudes given by: $A_a = 0$, $A_b = 5$ and $A_c = 0.5$.

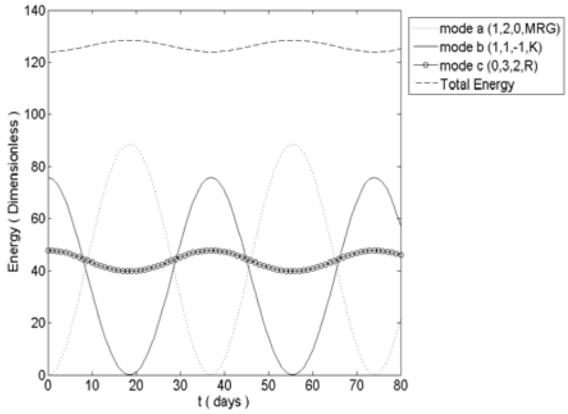

Fig. 1: Time evolution of the mode energies $c_j^2|A_j|^2$, j = a, b and c, for the resonant triad explored in this paper. The initial amplitudes used for the integration of equations (6) in this case are given by $A_a = 0$, $A_b = 5$ and $A_c = 0.5$.

Fig. 1 shows that in this resonant interaction the energy of the MRGW (mode a), which is the highest absolute frequency mode of this triad, always grows (or decays) at the expense of the remaining triad components. The barotropic Rossby mode in this interaction is the less energetically active member. Nevertheless, the interaction period is dependent on the initial amplitude of this mode (figures not shown). It is important to mention that even though the interaction period is dependent on the initial wave amplitudes, the initial mode amplitudes set in Fig. 1 reproduce typical magnitudes of observed atmospheric flow anomalies, with wind magnitudes of the order of 20m/s (Fig. 2). Thus, for the initial amplitudes reproducing typical magnitudes of atmospheric flow perturbations, Fig. 1 shows that the modes of the resonant triad studied here exchange energy on intraseasonal time-scales, with the interaction period being dependent on the initial energy and the way in which the initial energy is distributed among the triad components.

Fig. 2 shows the 1000hPa horizontal wind and geopotential fields at t = 0 associated with the solution of Fig. 1. The horizontal wind and geopotential fields shown in Fig. 2 are obtained by expansion (5) truncated in such a way to consider only the triad components of Fig. 1. One can notice in Fig. 2 that the barotropic Rossby wave in this triad interaction has a large projection onto middle and higher latitudes. This mode also shares some features in common with the Pacific South American (PSA) and the Pacific North American (PNA) teleconnection patterns. In fact, several studies (Ghil and Mo 1991; Rogers and van Loon 1982; Kidson 1988, 1991; Mo and Higgins 1998) have documented through

Principal Component Analysis patterns of the streamfunction characterized by a zonal wavenumber-3 structure in midlatitudes, with a larger amplitude over the Pacific-American sector, and an approximate phase opposition between the sub-tropics and midlatitudes. According to these observational studies, these patterns exhibit a pronounced intraseasonal variability.

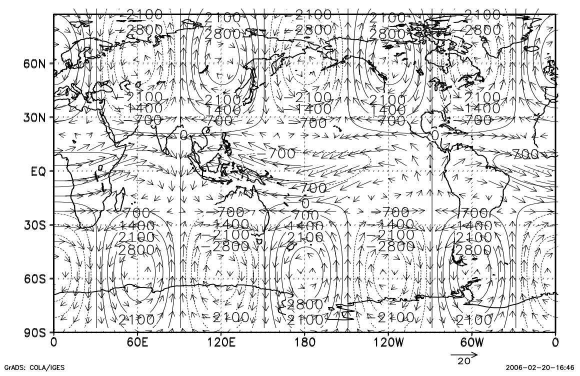

Fig. 2: Horizontal distribution of the wind (vector) and geopotential (contours) fields at p = 1000hPa associated with the solution shown in Fig. 1 at t = 0. The horizontal wind and geopotential fields are shown in m/s and m^2s^{-2}, respectively, using the scales in (3) for U = 5m/s, $\beta = 2{,}3 \times 10^{-11}\ m^{-1}s^{-1}$ and L = 10^6m.

On the other hand, the zonal wavenumber-1 Kelvin wave having the first baroclinic mode vertical structure (mode b) has several features in common with the observed Madden-Julian Oscillation (MJO) in the tropics. In fact, the equatorial MJO is characterized by a global zonal wavenumber-1 Kelvin wave structure and a vertical structure similar to that associated with the first baroclinic vertical eigenmode, having an eastward propagation speed of the order of 5m/s over strongly convective regions, such as the Eastern Indian and the Western Pacific Oceans, and of 15m/s out of these convective regions (Madden and Julian 1994; Hendon and Salby 1994; Wheeler and Hendon 2004). The OLR (outgoing long-wave radiation) anomalies associated with this oscillation have typically the strongest signal over the Western Pacific and the Eastern Indian Ocean sectors, showing a phase opposition of the signal between these regions, and practically disappear as the disturbance propagates away from these sectors. However, the circulation anomalies persist as the oscillation propagates throughout the entire globe and modulate the monsoon activities (Wang et al. 2005) as well as the activity and/or intensity of hurricanes (Maloney and Hartmann 1999).

Therefore, due to the possible relation of the triad modes analyzed here to observed features in the atmospheric circulation, the energy exchanges among these modes shown in Fig. 1 can also have a potential role on the tropics-extratropics interaction on intra-seasonal time-scales. The results suggest that the resonant coupling between the zonal wavenumber-3 barotropic Rossby wave having the n = 2 meridional mode structure and the zonal wavenumber-1 Kelvin wave with the first baroclinic mode vertical structure through the zonal wavenumber-2 Yanai wave having the first baroclinic mode vertical structure can be a possible mechanism of linkage between the MJO activity in the tropics and the equivalent barotropic midlatitude teleconnection patterns such as the PNA and the PSA patterns.

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